Lecture 8:
Producer Behavior

October 24, 2017

Overview

Course Administration

1. Problem Sets
   - Return Problem Set 6
   - Problem Set 6 answers posted
   - Problem Set 7 posted
2. Midterm
   - return at end of class
   - will post answers tomorrow
3. Elasticity memo: Drafts should be posted; comments due Sunday
4. Any questions?

Ripped from the Headlines

Afternoon

Finder	Presenter
Nathalie Grogan	Matthew Pickering
Hakim Jan	McCall Hopkin
Amanda Fins	Peter Aziz

Evening

Finder	Presenter
Dara Duratinsky	Elisa Walker
Leslie Zelenko	Ray Lazott
Shelbe Klebs	Hannah Seligman

Basics of Production

What is Production?

- Production $\equiv$ process of producing a good or service
- Final good $\equiv$ good bought by consumer
- Intermediate good $\equiv$ good bought by a firm to produce another good
- Production function $\equiv$ mathematical relationship between inputs and outputs

Simplifying Assumptions, 1 of 2

Why do we assume things? To make the problem manageable and look carefully at a limited number of factors.

Simplifying Assumptions, 1 of 2

Why do we assume things? To make the problem manageable and look carefully at a limited number of factors.

1. Firm produces a single good
2. Firm has already chosen what product it will produce
3. Firm's goal is to minimize cost
4. Firm uses only two inputs: capital and labor
5. In the short run, the firm can change only labor. In the long run the firm can change labor and capital

Simplifying Assumptions, 2 of 2

6. More inputs $\rightarrow$ more outputs
7. Production has diminishing marginal returns to capital and labor
8. An infinite amount of inputs sells at fixed prices
9. The firm has no budget constraint $\rightarrow$ very well-functioning capital market

The Production Function

$$Q = f(K, L)$$

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- Q is output
- K is capital
- L is labor
- $f()$ is a general function

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For example, $Q = K^{0.5}L^{0.5}$.

Parallels: Consumer and Producer Problems

What is the producer parallel of the utility function?

Consumer	Producer
Diminishing marginal utility	
max U s.t. budget constraint	
Utility function	
Indifference curves	
$MRS_{X,Y}$	
Price of consumption goods	
Budget Constraint	
Slope of budget constraint $= -\frac{P_X}{P_Y}$	
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Production in the Short Run

Measuring Changes in Production

1. Marginal product of $X \equiv$ additional output from an additional unit of input X (X is K or L), holding all other inputs fixed

$$MP_X = \frac{\Delta Q}{\Delta X} = \left(\frac{\partial Q}{\partial X}\right)$$

Measuring Changes in Production

1. Marginal product of $X \equiv$ additional output from an additional unit of input X (X is K or L), holding all other inputs fixed

$$MP_X = \frac{\Delta Q}{\Delta X} = \left(\frac{\partial Q}{\partial X}\right)$$

2. Average product of X

$$AP_X = \frac{Q}{X}$$

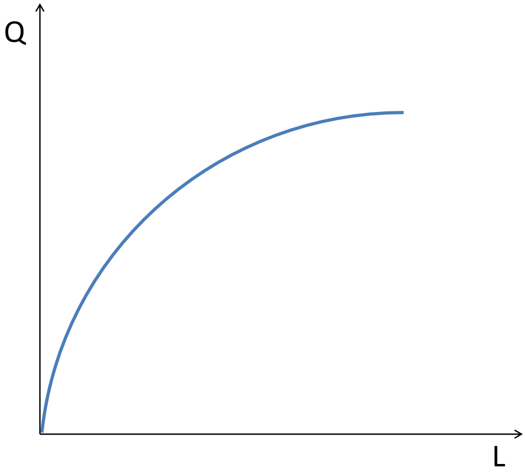
Measuring Changes in the Short Run

- Recall: We assume that in the short run K is fixed and L can change
- Suppose $K = 5$, and $Q = f(K, L)$
- Then the short run production function is

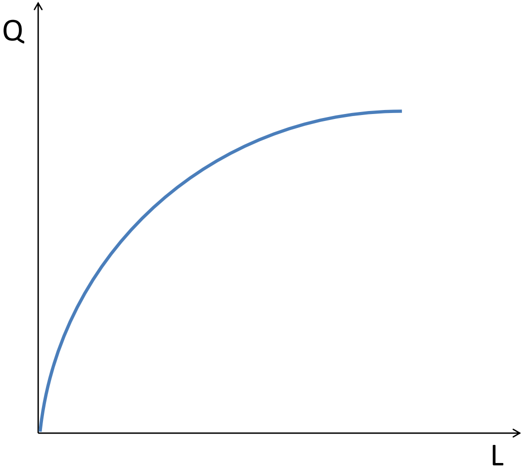
Measuring Changes in the Short Run

- Recall: We assume that in the short run K is fixed and L can change
- Suppose $K = 5$, and $Q = f(K, L)$
- Then the short run production function is $Q = f(5, L)$
- Recall that we assumed diminishing marginal product of labor
- Draw short-run output as a function of labor (Q on the y axis, L on the x axis)

Short-Run Production Function

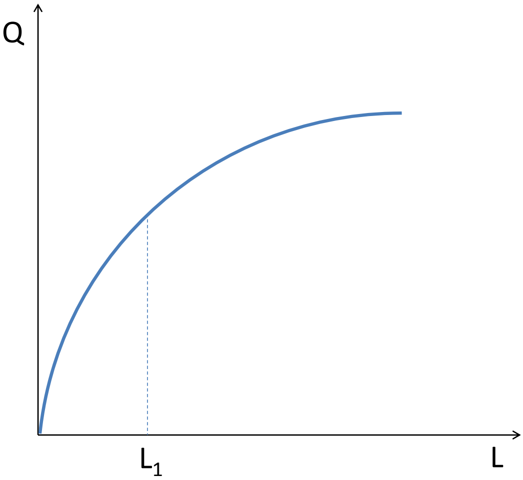


Finding the Marginal Product of Labor from the Production Function



Finding the Marginal Product of Labor from the Production Function

What is the marginal product of labor here?



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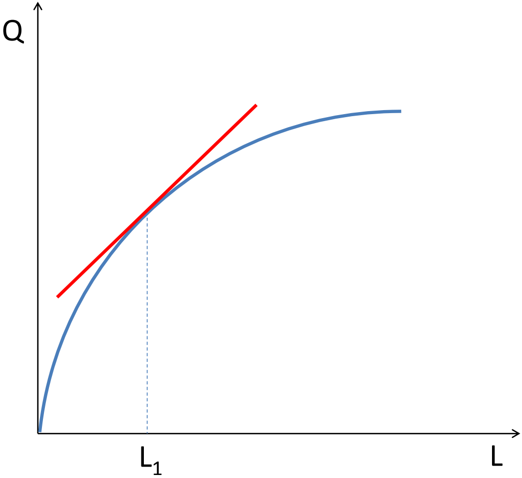

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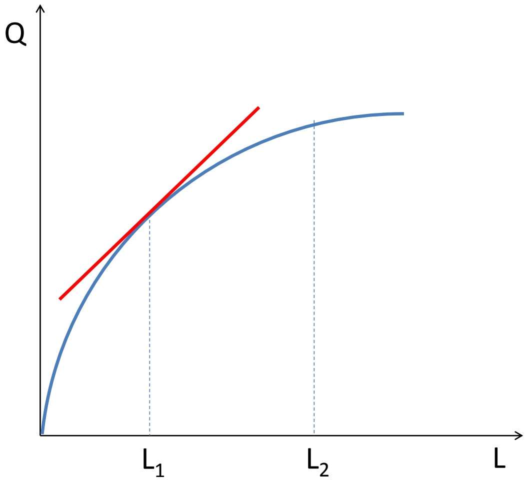

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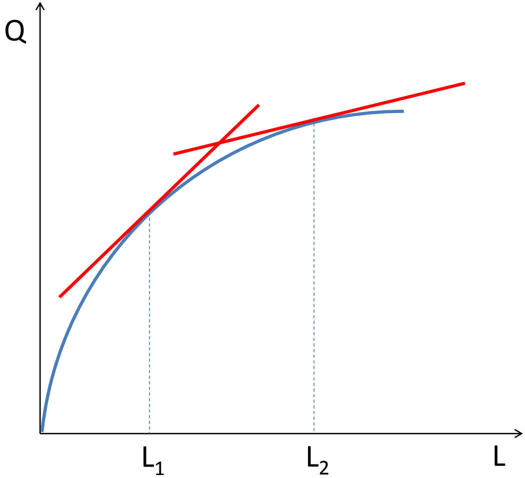

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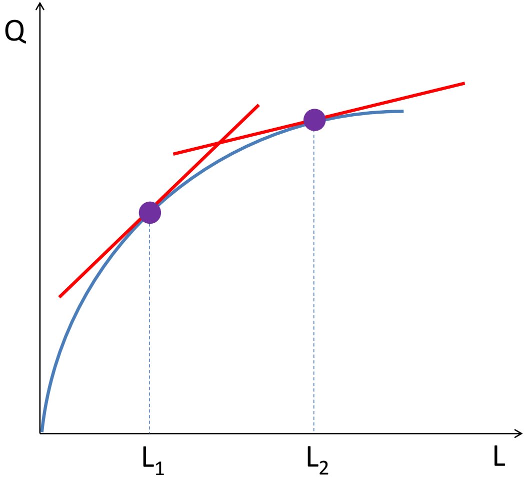
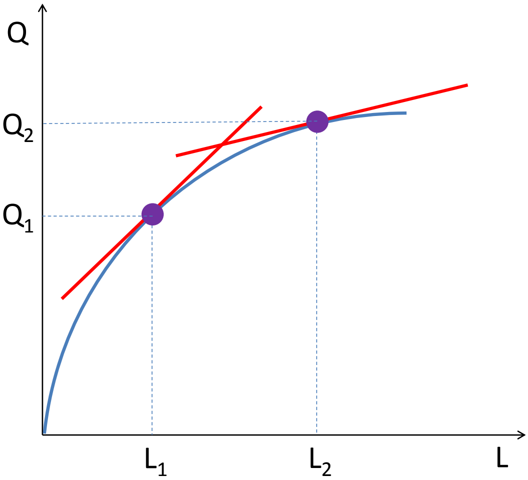


Finding the Marginal Product of Labor from the Production Function

So what does a graph of MP_L as a function of Q look like?



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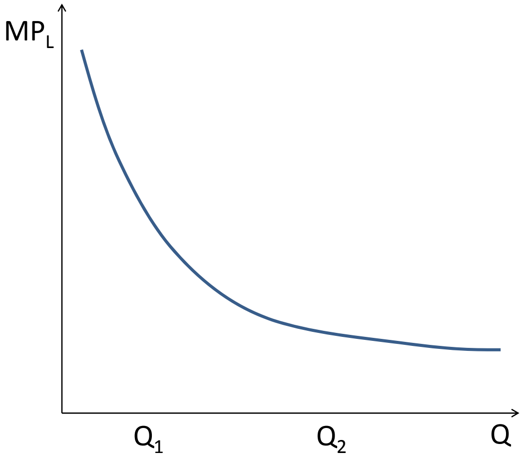

Parallels: Consumer and Producer Problems

What is the producer parallel of diminishing marginal utility?

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max U s.t. budget constraint	
Utility function	production function
Indifference curves	
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Production in the Long Run

Production in the Long Run

- In the long run, everything can change
- Diminishing returns are less of a problem, since you can add both capital and labor

Minimizing Costs

Firm's Problem

- Firm wants to minimize costs
- Subject to producing a given amount of output

Firm's Problem

- Firm wants to minimize costs
- Subject to producing a given amount of output
- It could always minimize costs by shutting down, but then no one is making any money

Parallels: Consumer and Producer Problems

What is the producer parallel of maximizing utility subject to a budget constraint?

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Diminishing marginal utility	diminishing marginal product
max U s.t. budget constraint	
Utility function	production function
Indifference curves	
$MRS_{X,Y}$	
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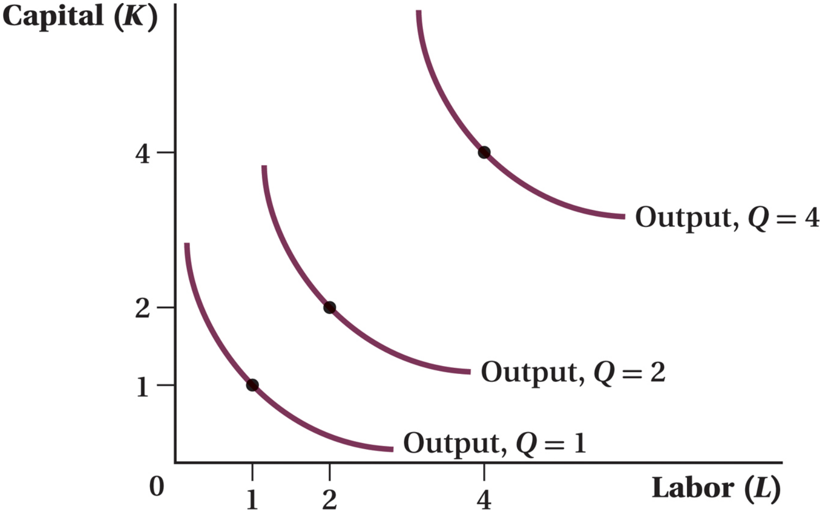
Isoquants

- "iso" $\equiv$ same
- "quant" for quantity
- All combinations of K and L that produce some level of Q
- Properties of isoquants, for a given production function
  - Further from the origin $\rightarrow$ more production
  - Cannot intersect
  - Convex to the origin

Short-Run Production Function

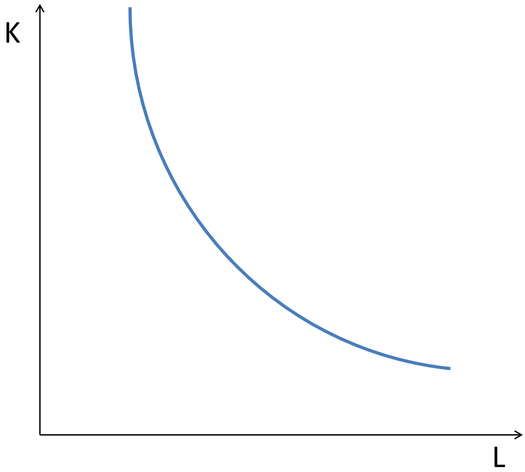


Marginal Rate of Technical Substitution

- $MRTS_{XY} \equiv$ slope of the isoquant
- Or, the rate at which firm can trade input L for input K, holding output constant

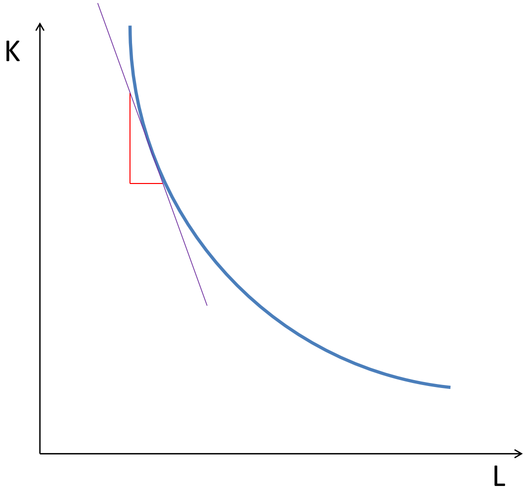
Marginal Rate of Technical Substitution Along an Isoquant

What Does the Shape of the Isoquant Tell Us About the Trade-off Between Capital and Labor?



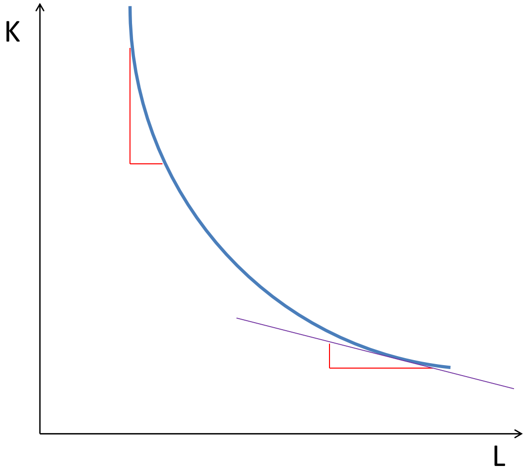
Marginal Rate of Technical Substitution Along an Isoquant

And at the Other End?



Marginal Rate of Technical Substitution Along an Isoquant

Diminishing Marginal Product in Action



Parallels: Consumer and Producer Problems

What is the producer parallel of indifference curve?

Consumer	Producer
Diminishing marginal utility	diminishing marginal product
max U s.t. budget constraint	min C s.t. producing $Q = a$
Utility function	production function
Indifference curves	
$MRS_{X,Y}$	
Price of consumption goods	
Budget Constraint	
Slope of budget constraint $= -\frac{P_X}{P_Y}$	
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Parallels: Consumer and Producer Problems

What is the producer parallel of the marginal rate of substitution?

Consumer	Producer
Diminishing marginal utility	diminishing marginal product
max U s.t. budget constraint	min C s.t. producing $Q = a$
Utility function	production function
Indifference curves	Isoquants
$MRS_{X,Y}$	
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Parallels: Consumer and Producer Problems

Isoquants and *MRTS*

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Input Substitutability and Complementarity

What Does it Mean for the Production Function?

What do the isoquants look like if

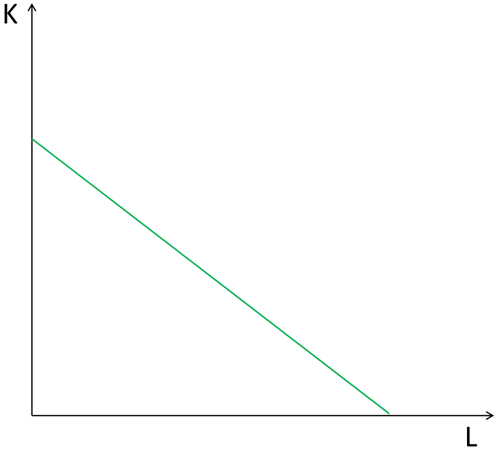
- inputs are perfect substitutes?
- inputs are perfect complements?

Isocost Lines

- Cost of capital is R: rental rate per period
- Cost of labor is W: wage rate per period
- For any cost C, the isocost line is $C = RK + WL$
- What's the slope of the isocost line?

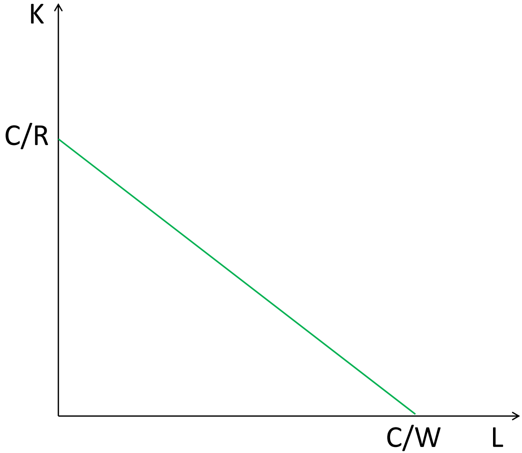
Drawing the Isocost Curve

What are the endpoints of the isocost curve?



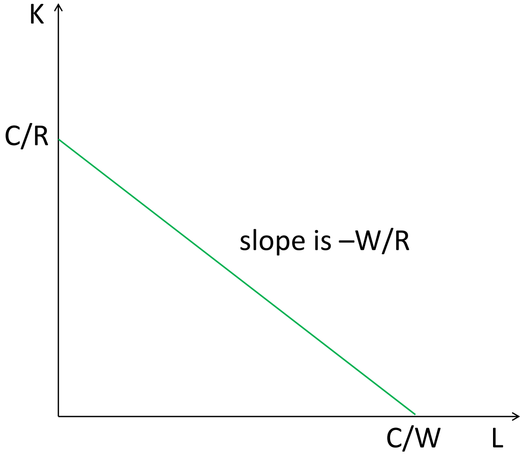
Drawing the Isocost Curve

Endpoints of the isocost curve



Drawing the Isocost Curve

Slope of the isocost curve

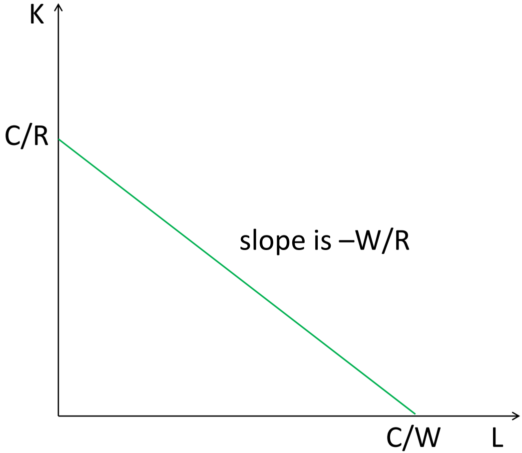


Implications of Isocost Line

- Slope of isocost line is the cost consequences of trading off one unit of K for L
- What if the price of K increases? decreases?
- Labor?

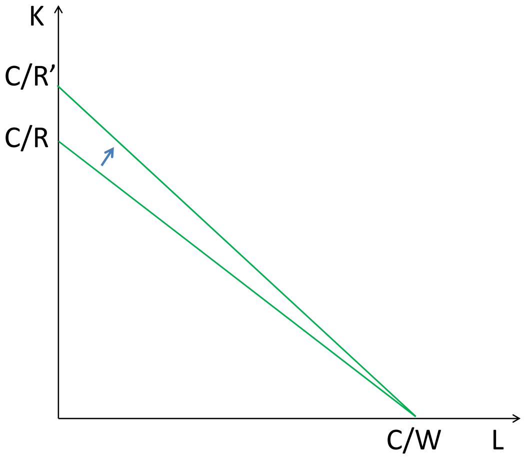
Changes to the Isocost Curve

What if the price of capital declines?



Changes to the Isocost Curve

The isocost curve twists



Parallels: Consumer and Producer Problems

What are the relevant producer prices?

Consumer	Producer
Diminishing marginal utility	diminishing marginal product
max U s.t. budget constraint	min C s.t. producing $Q = a$
Utility function	production function
Indifference curves	Isoquants
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Price of consumption goods	
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Parallels: Consumer and Producer Problems

What is the producer parallel of the budget constraint?

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Diminishing marginal utility	diminishing marginal product
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Utility function	production function
Indifference curves	Isoquants
$MRS_{X,Y}$	$MRTS_{LK}$
Price of consumption goods	$P_L = W$, $P_K = R$
Budget Constraint	
Slope of budget constraint $= -\frac{P_X}{P_Y}$	
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Indifference curves	Isoquants
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Budget Constraint	Isocost line
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Parallels: Consumer and Producer Problems

Budget constraint ≈ Isocost

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Price of consumption goods	$P_L = W$, $P_K = R$
Budget Constraint	Isocost line
Slope of budget constraint $= -\frac{P_X}{P_Y}$	Slope of isocost $= -\frac{W}{R}$
Optimality at $MRS_{XY} = \frac{P_X}{P_Y}$	
Income expansion path	

Finding Minimum Cost

- Firm wants to produce a given output at minimum cost
- A constrained minimization problem
- Constraint is that firm produces some level of output Q
    - Think of this as a given: $Q = a$
    - Consumer problem: income is given, we find maximum happiness
    - Producer problem: Q is given, and we find minimum cost
- Goal: what is the lowest cost at which it can produce that output?

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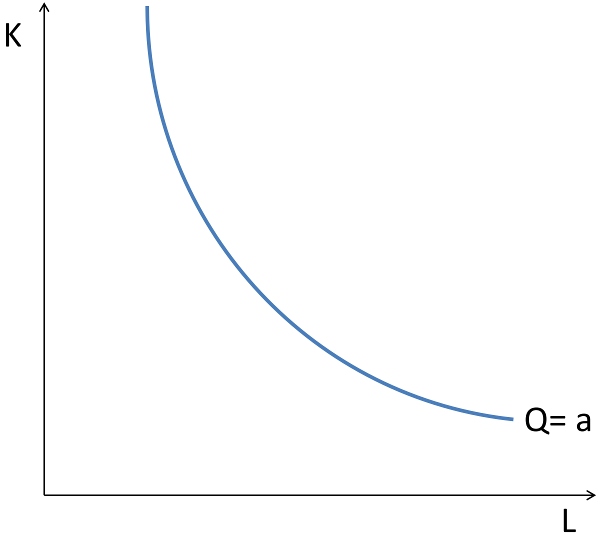
- Goal: what is the lowest cost at which it can produce that output?
- Cost minimization is necessary but not sufficient for profit maximization – more on this later

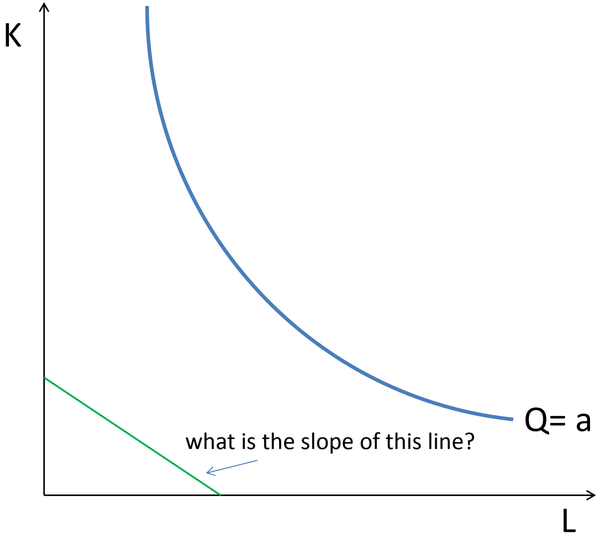
Cost Minimization in Pictures

How Can You Produce $Q = a$ at Minimum Cost?



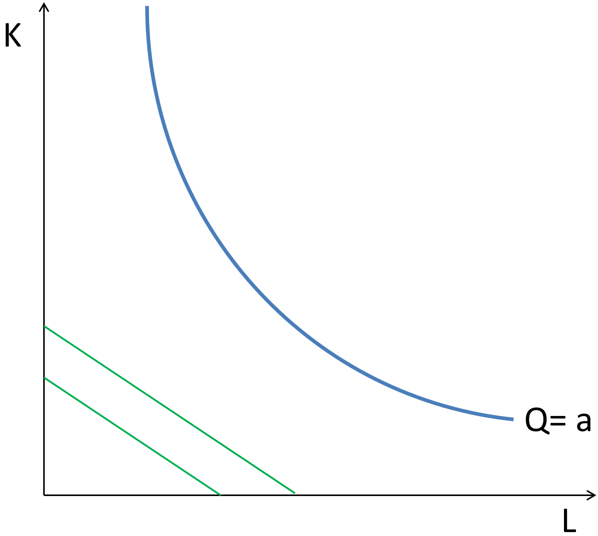
Cost Minimization in Pictures

Find the Slope of the Isocost Line



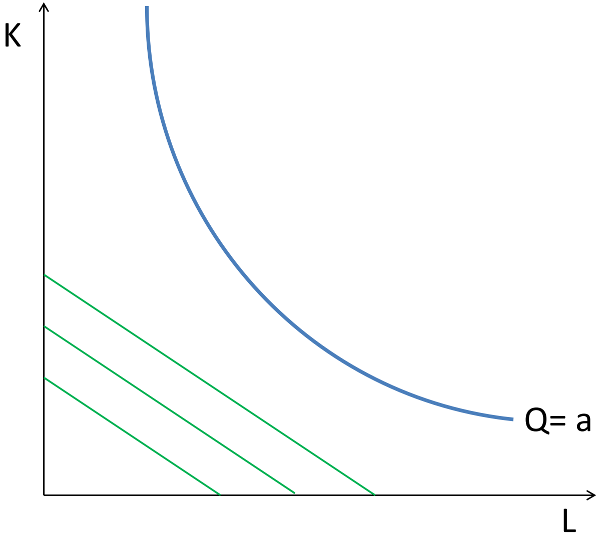
Cost Minimization in Pictures

Not Enough Inputs to Make *a*



Cost Minimization in Pictures

Still Not Enough



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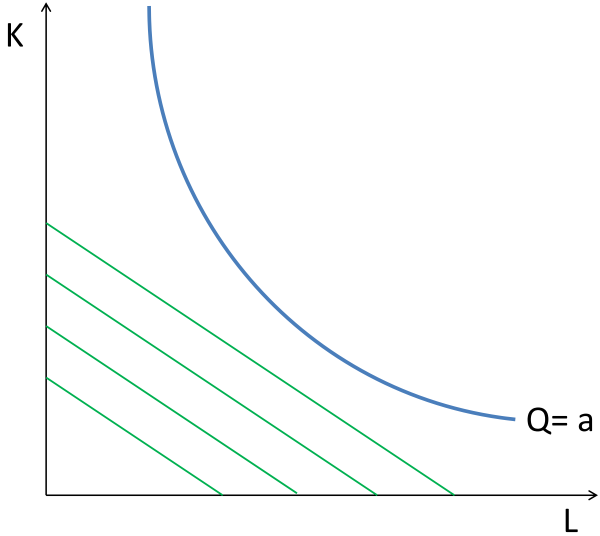
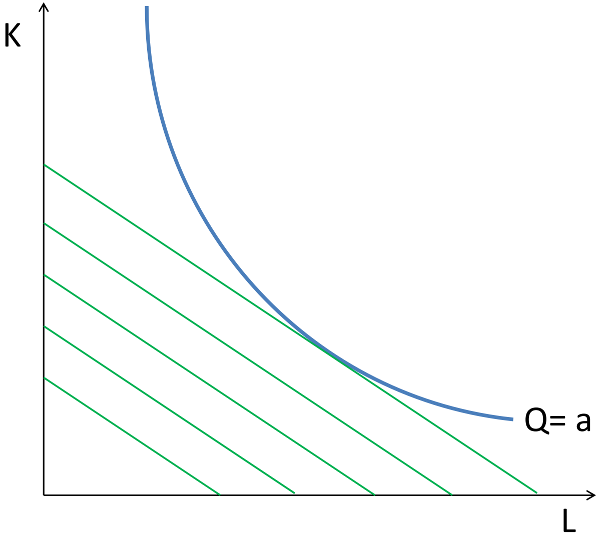


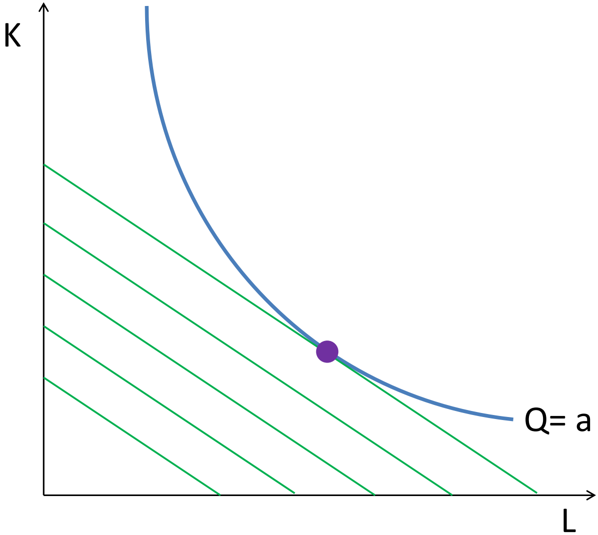
Cost Minimization in Pictures

Enough?



Cost Minimization in Pictures

The Optimal Combination of K and L



Conditions for Cost Minimization

- Occurs where isocost is tangent to isoquant
- Occurs when

$$-MRTS_{LK} = -\frac{P_L}{P_K}$$

$$-\frac{MP_L}{MP_K} = -\frac{W}{R}$$

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- Occurs where isocost is tangent to isoquant
- Occurs when

$$-MRTS_{LK} = -\frac{P_L}{P_K}$$

$$-\frac{MP_L}{MP_K} = -\frac{W}{R}$$

- More intuitively,

$$\frac{MP_L}{W} = \frac{MP_K}{R}$$

- Marginal product per dollar is equal

Parallels: Consumer and Producer Problems

What is the producer optimality condition?

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Diminishing marginal utility	diminishing marginal product
max U s.t. budget constraint	min C s.t. producing $Q = a$
Utility function	production function
Indifference curves	Isoquants
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Price of consumption goods	$P_L = W$, $P_K = R$
Budget Constraint	Isocost line
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Parallels: Consumer and Producer Problems

Think tangency!

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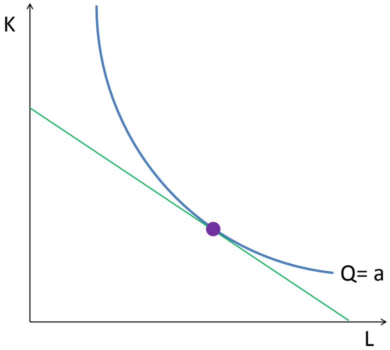
What if Input Prices Change?

- Price of labor increases, and price of capital decreases



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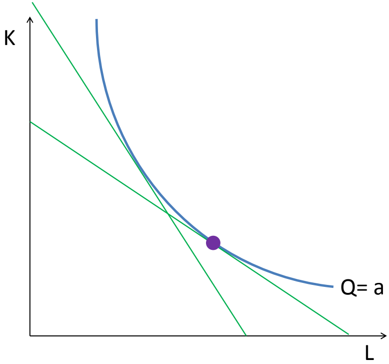
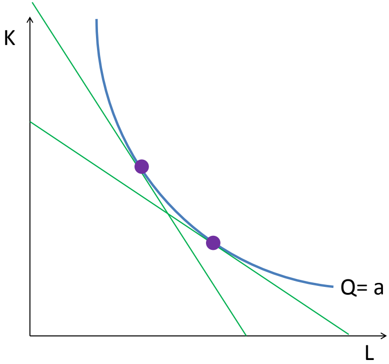


What if Input Prices Change?

- Price of labor increases, and price of capital decreases
- Firms adjust to use more of the less costly input



Minimize Costs on Your Own

A firm employs 25 workers ($W = \$10/\text{hour}$) and 5 units of capital ($R = \$20/\text{hour}$). At these levels, the marginal product of labor is 25, and the marginal product of capital is 30.

1. Is this firm minimizing costs?
2. If not, what changes should it make?
3. How does the answer to question 2 depend on the time frame of analysis?

Returns to Scale

Returns to Scale

Returns to Scale ≡ changes in output given a change in inputs.

- Suppose your production function is

$$Q = 5K + L$$

Returns to Scale

Returns to Scale $\equiv$ changes in output given a change in inputs.

- Suppose your production function is

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- Double inputs: $K' = 2K$, $L' = 2L$

Returns to Scale

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- Find new Q, call it Q', relative to old Q

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$$Q = 5K + L$$

- Double inputs: $K' = 2K$, $L' = 2L$
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$$\begin{aligned} Q' &= 5K' + L' \\ &= 5(2K) + (2L) \\ &= 2(5K + L) \\ &= 2Q \end{aligned}$$

Returns to Scale

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- Suppose your production function is

$$Q = 5K + L$$

- Double inputs: $K' = 2K$, $L' = 2L$
- Find new Q, call it Q', relative to old Q

$$\begin{aligned} Q' &= 5K' + L' \\ &= 5(2K) + (2L) \\ &= 2(5K + L) \\ &= 2Q \end{aligned}$$

We call this constant returns to scale.

Flavors of Returns to Scale

- constant $\rightarrow$ outputs increase proportionately with inputs
  - double inputs, double outputs

Flavors of Returns to Scale

- constant $\rightarrow$ outputs increase proportionately with inputs
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- increasing $\rightarrow$ outputs increase more than proportionately with inputs
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In general, put in inputs, find Q.
Double the inputs, find Q'. Is $Q' = 2Q$? $Q' > 2Q$? $Q' < 2Q$?

What Drives Returns to Scale?

- Increasing returns
  - Fixed costs
  - Learning by doing – if the firm gets bigger and better at production by producing
- Decreasing returns
  - Regulation
  - Limited low cost/high quality inputs (violates one of our assumptions)

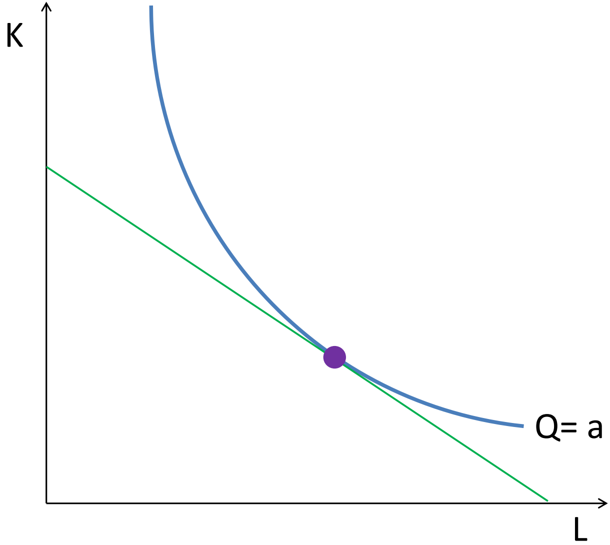
Expansion Path

How Does Production Change at Different Levels of Q?

- We know how to find the firm's ideal inputs given Q
- Now we repeat this exercise for a variety of different Qs
    - Each optimal K and L will be where an isoquant is tangent to an isocost line
    - $MRTS_{LK}$ will be the same at each point
- Call this optimal (L, K) for each Q the expansion path
- And we can draw a total cost curve – with different axes

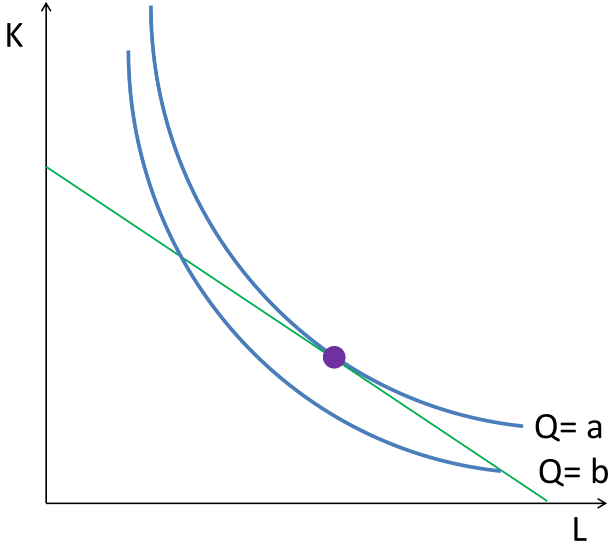
Drawing a Total Cost Curve

Recall Our Previous Optimum. What if the firm wants to produce $b < a$?



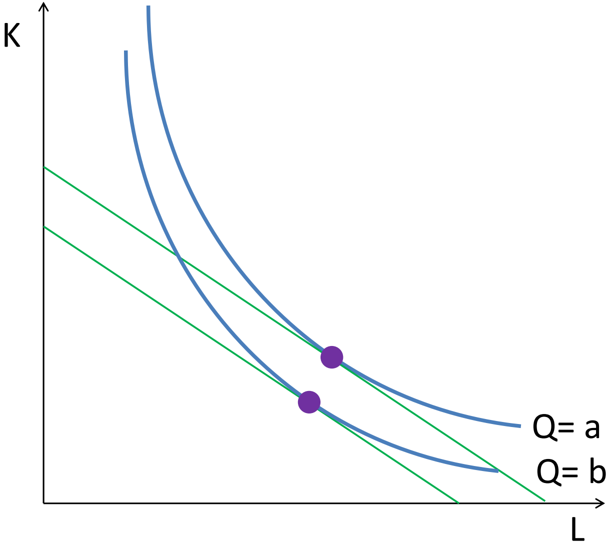
Drawing a Total Cost Curve

Recall Our Previous Optimum. What if the firm wants to produce $b < a$?



Drawing a Total Cost Curve

What is optimum (L, K)if it wants to make $c < b$?



Drawing a Total Cost Curve

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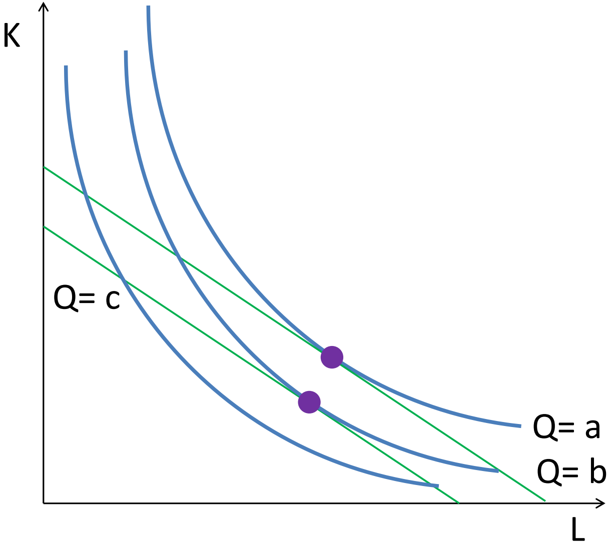

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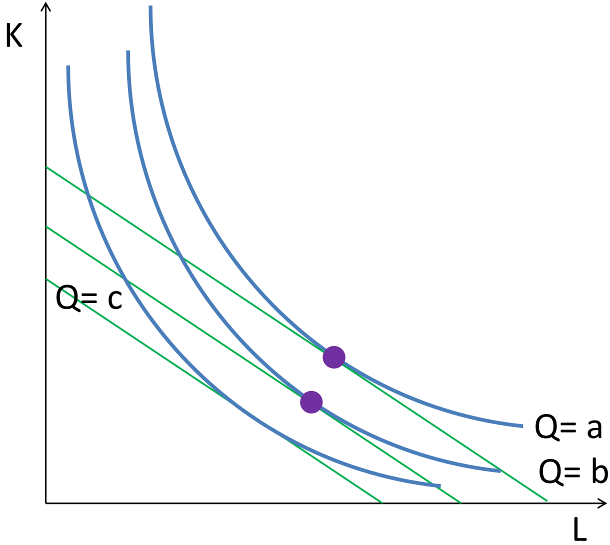
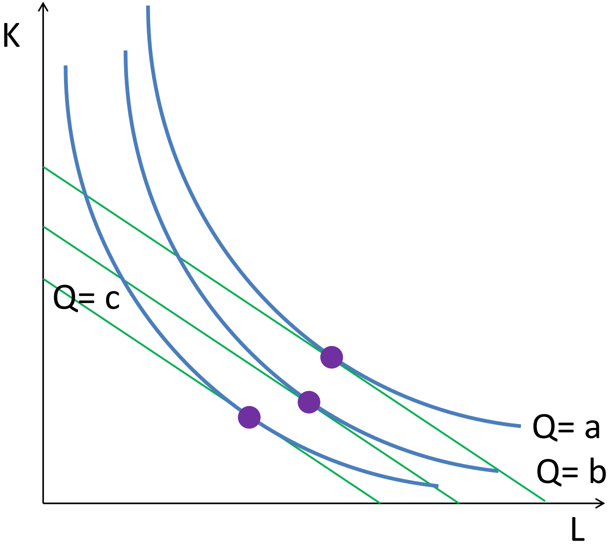


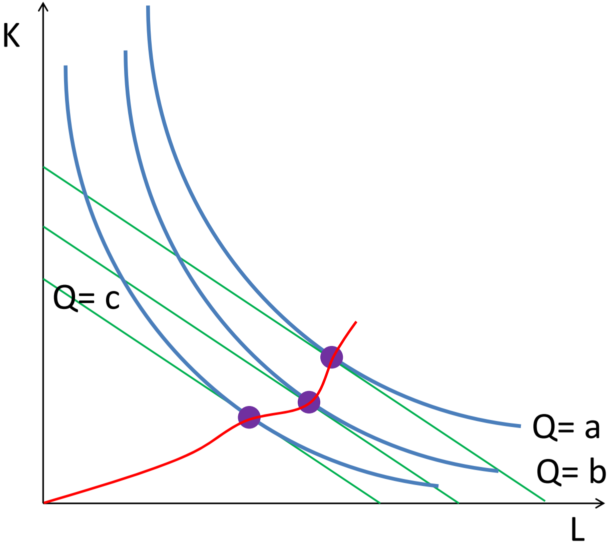
Drawing a Total Cost Curve

Optimum!



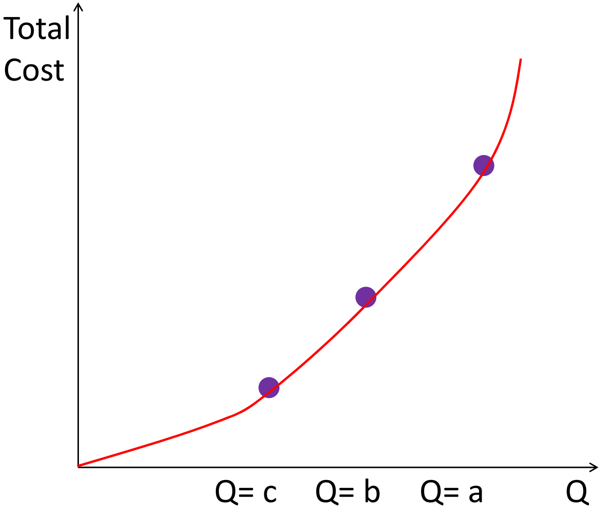
Drawing a Total Cost Curve

Expansion Path



Drawing a Total Cost Curve

Total Cost



Parallels: Consumer and Producer Problems

What is the producer parallel of the income expansion path?

Consumer	Producer
Diminishing marginal utility	Dimin. MP of L, K constant
Utility function	Production function
max U s.t. budget constraint	min cost s.t. production of Q
Indifference curves	Isoquants
$MRS_{X,Y}$	$MRTS_{L,K}$
Price of consumption goods	$P_L = W$, $P_K = R$
Budget Constraint	Isocost curve
Slope of budget constraint $= -\frac{P_X}{P_Y}$	Slope of isocost curve $= -\frac{P_L}{P_K}$
Optimality at $MRS_{XY} = \frac{P_X}{P_Y}$	$MRTS = \frac{W}{R}$
Income expansion path	

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Optimality at $MRS_{XY} = \frac{P_X}{P_Y}$	$MRTS = \frac{W}{R}$
Income expansion path	Expansion path

Recap of Today

- Production Assumptions and Basics
- Production in the Short Run
- Production in the Long Run
- Cost Minimization Problem
- Returns to Scale
- Expansion Path and Total Cost
- Technological Change (time does not permit)

Midterm Results Distribution

Score	Afternoon	Evening	Both
30-39	1	0	1
40-49	2	1	3
50-59	4	0	4
60-69	5	4	9
70-79	6	9	15
80-89	5	9	14
90-100	4	6	10
Mean	71.4	80.1	75.9
Std. dev.	16.5	11.2	14.6

Curve yields grades

- 90 to 100 A
- 82 to 90 A-
- 72 to 82 B+
- 65 to 72 B
- 58 to 65 B-
- 48 to 58 C+
- 40 to 48 C
- 30 to 40 C-

- If you are on the border of a letter grade, I round up.
- If you got an A and are willing to volunteer to help a student, send me an email
- If you got a B or below and would like help from a student volunteer, send me an email

Next Class

- Turn in Problem Set 7
- Read GLS, Chapter 7