

Problem Set 9

On what and how to submit

- For this and all future problem sets, questions are from the “Problems” section of the questions at the end of the chapter.
- Due before Lecture 11.
- Name the file “ps09_[lastname].pdf”. For example, my file would be “ps09_brooks.pdf”.
- Turn in via this [google survey](#).
- Make sure your name is at the top of the submission.
- You do not need to type your submission. Any **legible** submission in pdf format is ok. For example, you can write the problem set with hand-drawn graphs, take a picture, make a pdf, and submit the pdf.

1. GLS Chapter 7, Question 14 (second edition, question 12)

There are many different ways to get to the correct answers in this problem. Here I outline one method, but this is not the only one.

Q	TC	FC	VC	MC	AVC	AFC	ATC
0	55	55	0	0	0	0	n/a
1	$17+55 = 72$	55	17	$=VC=17$	$17/1 = 17$	$55/1 = 55$	$72/1 = 72$
2	$72+15 = 87$	55	$87-55 = 32$	15	$32/2=16$	$55/2 = 27.5$	$87/2 = 43.5$
3	101	55	$101-55 = 46$	$46-32 = 14$	$46/3=15.3$	$55/3 = 18.3$	$101/3 = 33.6$
4	$55+58=113$	55	$14.5*4 = 58$	$113-101=12$	14.5	$55/4 = 13.75$	$113/4=28.25$
5	122	$122-67=55$	67	$122-113=9$	$67/5=13.4$	11	$122/5 = 24.4$
6	$21*6 = 126$	55	71	4	$71/6 = 11.8$	$55/6 = 9.2$	21

2. GLS Chapter 7, Question 16 (second edition, question 14)

Hint: remember the condition from Chapter 6 that must hold when the firm is minimizing costs.

(a) Find the total cost of producing 60 units of output.

The firm should produce this output at minimum cost. When the firm is producing at minimum cost, it must be true that

$$\frac{MP_L}{MP_K} = \frac{W}{R}$$

From what's given in this problem, we can rewrite this as

$$\frac{2K}{2L} = \frac{200}{100}$$

This becomes $K = 2L$.

If the firm is producing 60 units, it must also be the case that $60 = 2KL$, from the production function.

We can combine these two functions to solve for K and L . There are many ways to do this. I did

$$\begin{aligned} 60 &= 2KL \\ 60 &= 2(2L)L \\ 15 &= L^2 \\ L &= \sqrt{15} \end{aligned}$$

If we know L , we can then find K : $K = 2L = 2\sqrt{15}$.

Given the optimal K and L , we can now solve for the cost of using these inputs: $C = WL + RK$. Plugging in, we find $C = 200\sqrt{15} + 100(2)\sqrt{15} = 400\sqrt{15}$.

(b) Write the above, but with a general q , rather than an output of 60.

We start by saying that $q = 2KL$. From our finding in part (a), we know that when the firm is minimizing cost it must be true that $K = 2L$. Therefore, we can rewrite the production equation as $q = 2(2L)L = 4L^2$, or $L = \sqrt{\frac{q}{4}}$. Accordingly, $K = 2\sqrt{\frac{q}{4}} = 2\frac{\sqrt{q}}{2} = \sqrt{q}$.

Given these functions for L and K (note that each is expressed only in numbers and q), we

can rewrite a more general cost function:

$$\begin{aligned}C &= WL + RK \\&= W\sqrt{\frac{q}{4}} + R\sqrt{q} \\&= 200\sqrt{\frac{q}{4}} + 100\sqrt{q} \\&= 100\sqrt{q} + 100\sqrt{q} \\&= 200\sqrt{q}\end{aligned}$$

(c) Find average cost function.

$$\begin{aligned}\text{Average cost} &= \frac{\text{total cost}}{\text{total units}} \\&= \frac{200\sqrt{q}}{q} \\&= \frac{200}{\sqrt{q}}\end{aligned}$$

3. GLS Chapter 7, Question 21 (second edition, question 19)

(a) Find Peter's long run average cost.

$$\begin{aligned}\text{LAC} &= \frac{\text{LTC}}{Q} \\&= \frac{20,000Q - 200Q^2 + Q^3}{Q} \\&= 20,000 - 200Q + Q^2\end{aligned}$$

(b) Find Q where LAC is at a minimum.

We know that the long-run marginal cost curve crosses the long-run average cost curve at its minimum. Given that, the Q that solves for the intersection of the two curves is the Q at the minimum of the long-run average cost curve.

$$\begin{aligned}
\text{LMC} &= \text{LAC} \\
20,000 - 400Q + 3Q^2 &= 20,000 - 200Q + Q^2 \\
2Q^2 &= 200Q \\
Q &= 100
\end{aligned}$$

(c) Lowest possible average cost of production?

The lowest possible average cost is at a Q of 100. We know that long-run average cost is $= 20,000 - 200Q + Q^2$. Plugging in, we can write $\text{LAC} = 20,000 - 200(100) + 100^2 = 10,000$.

(d) Over what range of output are there economies of scale? diseconomies of scale?

The long-run average cost curve slopes downward until a Q of 100, so this section of the cost function yields economies of scale. After $Q = 100$, the cost curve slopes upward, and the firm experiences diseconomies of scale.

4. Sunk Costs and Opportunity Costs

Choose a firm. It should be a real firm, and any type of firm is fine.

(a) Name an input that is a sunk cost for this firm, explaining why.

(b) Describe the opportunity costs for two additional inputs, explaining why.

Less than one typed page should be more than sufficient to answer this question.

Any reasonably argued answer here is ok, as long as it refers to the correct definitions of sunk costs and opportunity costs.