Lecture 11:
Synthetic Control, or, Matching 2 of 2

April 2, 2025

Course Administration

1. Quantitative summary feedback posted
2. Workshop is next week
   - post material by Sunday midnight
   - post feedback for classmates before class
   - can agree on later within-group deadline as you like
3. Instructions for presentations posted

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4. Instructions for final paper posted – Lec. 9
5. Please come see me about your replication paper
6. Presentations April 16 and 23
7. Paper due April 28, 5 pm
8. Any other issues?

How to Present Things You Want to Compare

Original Table

	Treated	Untreated
Hats	6	5
Coats	8	6

Replication Table

	Treated	Untreated
Hats	6.2	5.1
Coats	8.1	6.1

How to Present Things You Want to Compare

Original Table with Your Work

	Original		Replication	
	Treated	Untreated	Treated	Untreated
Hats	6	5	6.2	5.1
Coats	8	6	8.1	6.1

Even Better

	Original		Replication		Percent Diff.	
	Treated	Untreated	Treated	Untreated	Treated	Untreated
Hats	6	5	6.2	5.1	0.03	0.02
Coats	8	6	8.1	6.1	0.10	0.02

Today

Synthetic control how-to

1. Overview
2. Set-up
3. Goal
4. Convex hull
5. Inference
6. Estimation
7. Example

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Synthetic control example

1. Research question
2. Outcomes
3. Estimation strategy
4. Results
5. What did you think?

Overview of Synthetic Control

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  - compare treated state to
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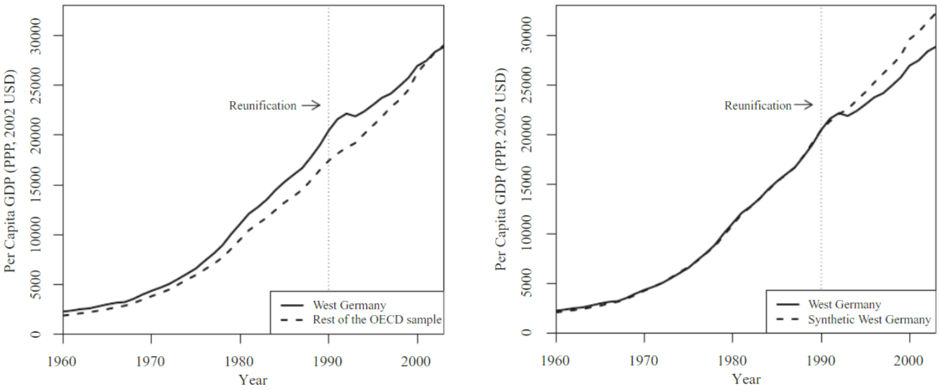
- This doesn't fix the small sample problem, but we use different inference methods

Basic Intuition: Impact of German Reunification

FIGURE 1 Trends in per Capita GDP: West Germany versus Rest of the OECD Sample

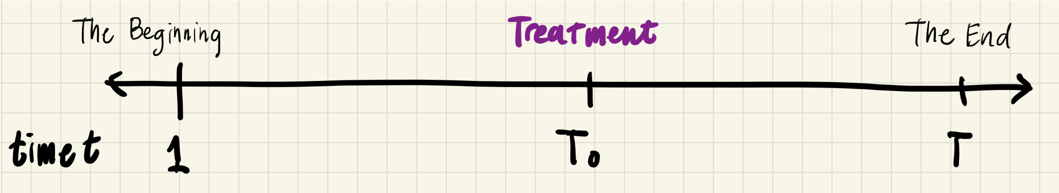
FIGURE 2 Trends in per Capita GDP: West Germany versus Synthetic West Germany



Abadie, Diamond and Hainmueller, "Comparative Politics and the Synthetic Control Method," 2015. [link]

Set-up

Time in Synthetic Control



- t is time, $t \in \{1, ..., T_0, ..., T\}$.
- Treatment occurs after T_0.
- We look for effects starting in $T_0 + 1$
  - there are T_0 pre-intervention periods, $\{1, ..., T_0\}$
  - there are T_1 post-intervention periods, $\{T_0 + 1..., T\}$
  - total $T = T_0 + T_1$

Units in Synthetic Control

- Each unit is denoted i
- There are $J+1$ total units, $i \in \{1..., J+1\}$
- Unit 1 is treated,
- Units $\{2, 3, ..., J+1\}$ are not treated
- $\rightarrow J$ untreated observations
- Call untreated observations the "donor pool"

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Many of these are not unique to synthetic control

Outcomes and Set-up for Synthetic Control

Notation

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- But! Problem: We only observe Y_{it}^{I}

Goal of Estimation

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Trick is to find w_j

Estimation

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  - $\sum_{j=1}^{J} \epsilon_i^2$
  - In matrix language that $\epsilon' \epsilon$
- In SC, we choose weights to minimize the difference between
  - the treated covariates and pre-treatment outcomes and
  - the donor pool's covariates and pre-treatment outcomes
- But remember that our optimal weights have **no** time dimension

Define What We Want To Match

We want X_0 to be like X_1

- X_1 is
  - pre-treatment covariates Z_1 and
  - pre-treatment outcomes $Y_{1,t<T_0}$ for the treated unit

$$X_1 \equiv \{Z_1, Y_{1,t<T_0}\}$$

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- Choose weights $\{w_2, w_2, \ldots, w_{J+1}\} \in W$ to minimize

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- We have one weight for each of the J donor pool observations

What we are minimizing

Remember, Z are covariates, Y are pre-treatment outcomes

$$X_1 = \begin{pmatrix} Z_{11} \\ Z_{12} \\ \vdots \\ Z_{1r} \\ Y_{11} \\ Y_{12} \\ \vdots \\ Y_{1T_0} \end{pmatrix},$$

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  - of as many numbers as you have Z and pre-treatment outcomes $Y_{t<T_0}$
- $\rightarrow$ final choice: how to weight to give to each difference in pre-treatment covariate or outcome so you can minimize

Choosing the Weights for Covariates

- We find donor pool weights w_j **conditional** on a decision about how much importance to give to each element of X
- Specifically, each element of X gets a weight v_k
- A final frontier of this estimation

Example

Suppose that there is one covariate and two pre-treatment observations. The matrix looks like this

$$Z_1 = 5, Z_0 = \begin{pmatrix} 0 & 5 \end{pmatrix}, Y_{1,t<T_0} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, Y_{0,t<T_0} = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$$

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$$||X_1 - X_0 W|| = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 & 5 \\ 2 & 5 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0w_1 + 5w_2 \\ 2w_1 + 5w_2 \\ 3w_1 + 8w_2 \end{pmatrix} = \begin{pmatrix} 5 - (0w_1 + 5w_2) \\ 2 - (2w_1 + 5w_2) \\ 3 - (3w_1 + 8w_2) \end{pmatrix}$$

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- We do know how to choose donor weights to minimize one number
- $\rightarrow$ make the vector one number by adding up its parts
- Have to decide how much we care about different parts of the diversion from the treated outcome to add them up
- The sum of all the parts of the vector is the mean squared error of the estimate
    - $\text{MSE} = ||X_1 - X_0W||v$, where v is yet another weighting matrix

How do you choose v?

A variety of options

- Give equal weight to all elements of X
- So that the pre-intervention difference in Y is minimized
- To minimize error in the final estimation (Abadie et al do this in another paper)
- Cross-validation in Germany paper:
  - find W for the first half of the pre-treatment era
  - choose v such that $||X_1 - X_0W||v$ is minimized in the second half of the pre-treatment period
  - if there are multiple possible W, you can see which one gives the lowest MSPE in the second pre-treatment period

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Note that $||X_1 - X_0W||v$ is the Mean Squared Prediction Error: MSPE

Re-capping Assumptions

- No effect of treatment on the untreated
- The treated unit would have had the untreated outcome in the absence of treatment
- Treated observation is in the convex hull of the donor pool

The Necessity of a Convex Hull

What is a Convex Hull?

- Key requirement
  - X_1 is in the convex hull of X_0
  - In words: the treated outcome's matching variables are in the convex hull of all donor pool Y and pre-treatment Z

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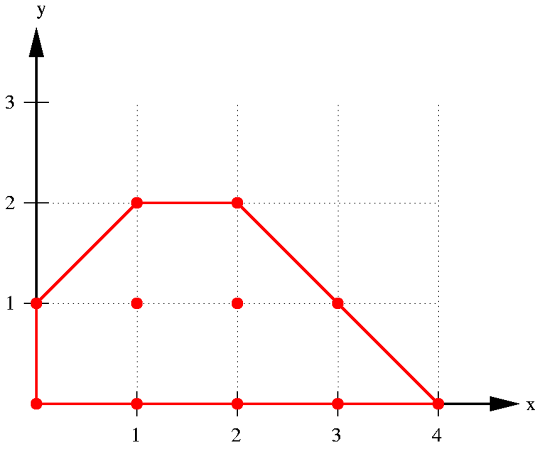
  - If you are inside the triangle, you are in the convex hull
  - If you are outside the triangle, you are outside the convex hull

Convex Hull Example



- Red dots are the set of points $x \in X$
- The red line is the outline of the convex hull
- Is $(2, 1)$ inside the convex hull?

Note: source is http://en.wikipedia.org/wiki/File:Convex_hull.png

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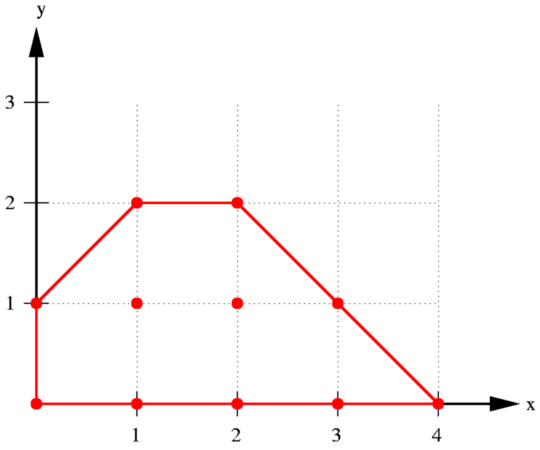

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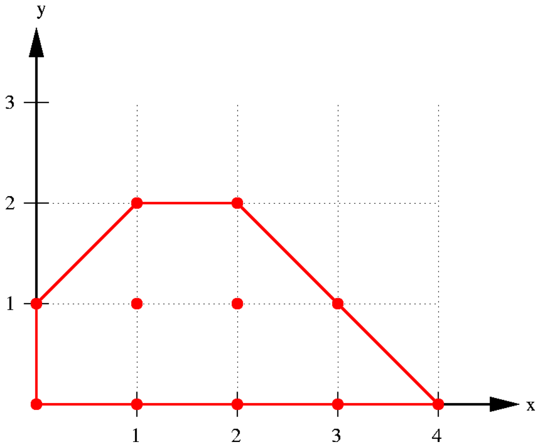

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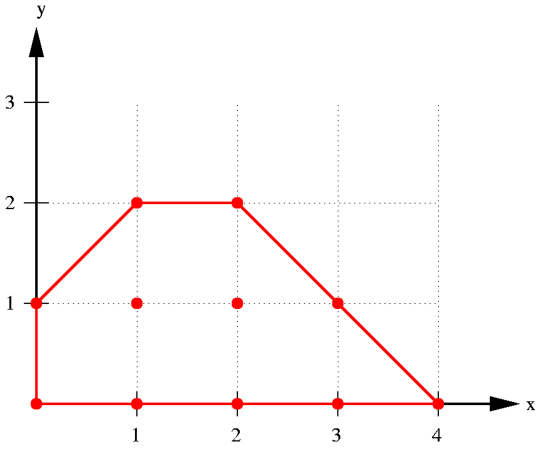

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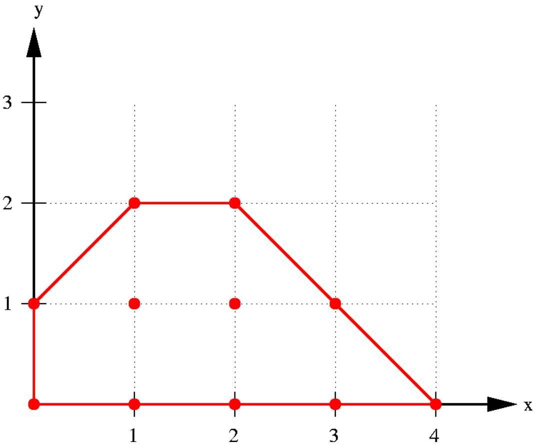

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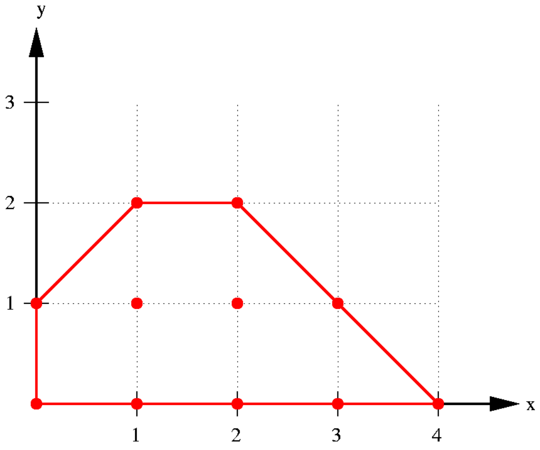
Convex Hull Example



- Red dots are the set of points $x \in X$
- The red line is the outline of the convex hull
- Is $(2,1)$ inside the convex hull? yes
- If $(2,1)$ is the treated point you can use a synthetic control
- Is $(3,3)$ inside the convex hull? no
- If $(3,3)$ is the treated point you **cannot** use a synthetic control

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- Sufficient condition for having donor pool observations in the convex hull
  - the "number of pre-intervention periods is large relative to the scale of the transitory shocks"
- Assuming that the donor pool lies in the convex hull is equivalent to assuming

$$X_1 - \sum_{j=2}^{J+1} w_j X_0 \equiv 0$$

  in words: pre-treatment outcome for treated observation can be re-created via a weighted average of untreated observations
- The convex hull assumption is sort of testable

Statistical Inference

Deciding Whether the Result is More Than Random Chance

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  - regression coefficient, which approximates parameter of interest
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- How do we make statistical inference in the case of synthetic control?

Thank you “statistical inference” in Wikipedia.

Statistical Interference in Synthetic Control

- A very open question
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Statistical Interference in Synthetic Control

- A very open question
- Check back in 5 to 10 years
- But we have some interim things we can do

One Way to Evaluate Error in Synthetic Control

RMSPE: Root Mean Squared Prediction Error

$$\text{RMSPE} = \left(\frac{1}{T_0} \sum_{t=1}^{T_0} \left(Y_{1t} - \sum_{j=2}^{J+1} w_j^* Y_{jt} \right)^2 \right)^{1/2}$$

This equation in words

- for each period before the treatment, $t < T_0$
- find squared difference between treated outcome, Y_{1t} , and synthetic control, $\sum_{j=2}^{J+1} w_j^* Y_{jt}$
- add up for all pre-treatment years and take square root

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- Then a big value for post / pre treatment RMSPE may be useful

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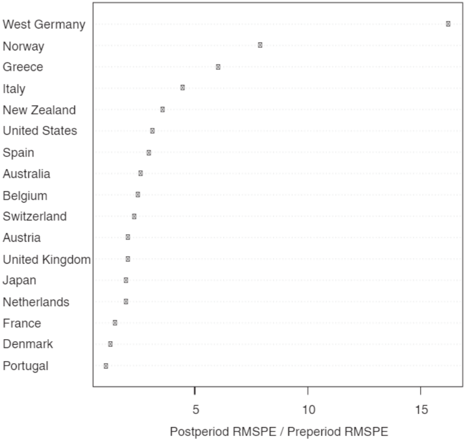
2. In space placebo
   - Pretend that another observation is the treated one
   - Do the synthetic control
   - Is post-treatment RMSPE for treated obs bigger than for other obs?

In Space Placebo

FIGURE 5 Ratio of Postreunification RMSPE to Prereunification RMSPE: West Germany and Control Countries



- Do synthetic control method for each country as if it were treated
- Find pre- and post-treatment RMSPE
- This is goodness of fit before – what you want
- And badness of fit afterward – what you see if there is an effect
- Chart reports this ratio
- West Germany is an outlier

California Tobacco Tax Example

Tobacco tax in California

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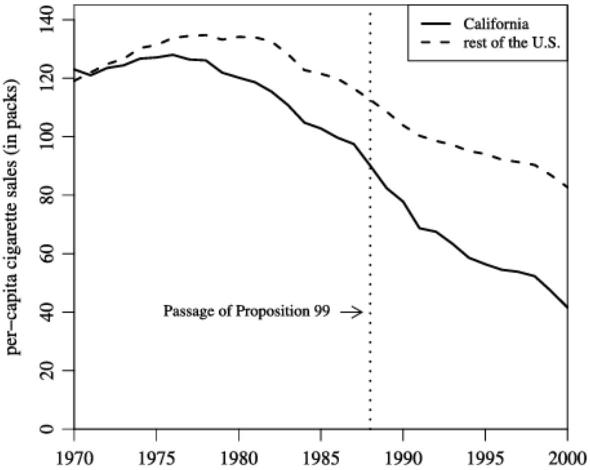
- Who is in the synthetic control?
- Utah (0.33), Nevada (0.23), Montana (0.22), Colorado (0.16), CT (0.07)

Abadie, Diamond and Hainmueller, 2010

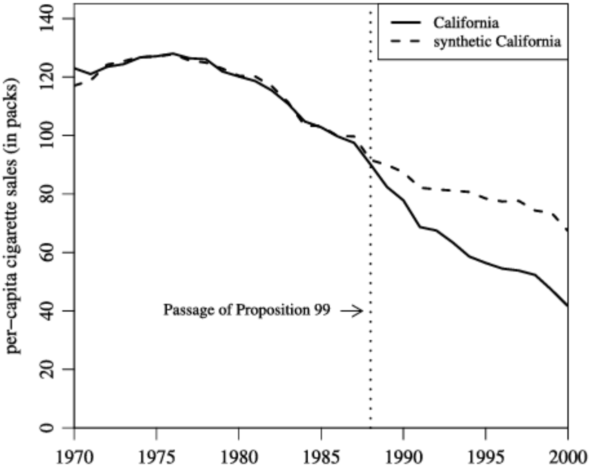
Things You Should Expect in a Synthetic Control Paper

1. Comparison without synthetic control
2. Comparison with synthetic control
3. Covariates with and without synthetic control
4. And sometimes even covariates on which we don't match
5. Some kind of inference

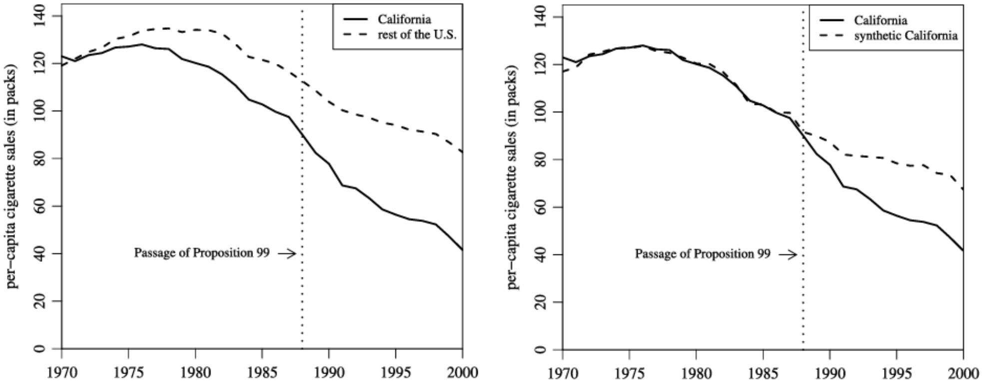
Prop. 99: Comparison without Synthetic Control



Prop. 99: Comparison With Synthetic Control



Prop. 99: Both Comparisons



Prop. 99: Covariates With Synthetic Control

Table 1. Cigarette sales predictor means

Variables	California Real	California Synthetic	Average of 38 control states
Ln(GDP per capita)	10.08	9.86	9.86
Percent aged 15–24	17.40	17.40	17.29
Retail price	89.42	89.41	87.27
Beer consumption per capita	24.28	24.20	23.75
Cigarette sales per capita 1988	90.10	91.62	114.20
Cigarette sales per capita 1980	120.20	120.43	136.58
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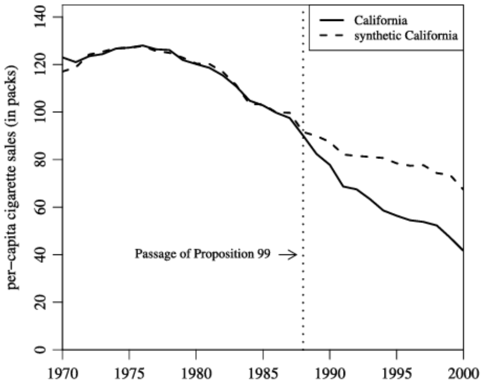
These are targeted covariates – we can also compare untargeted covariates

Prop. 99: Starting Inference



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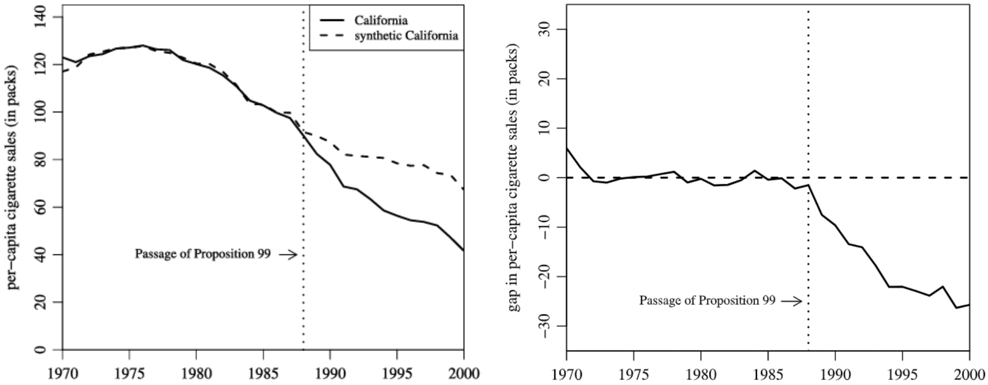
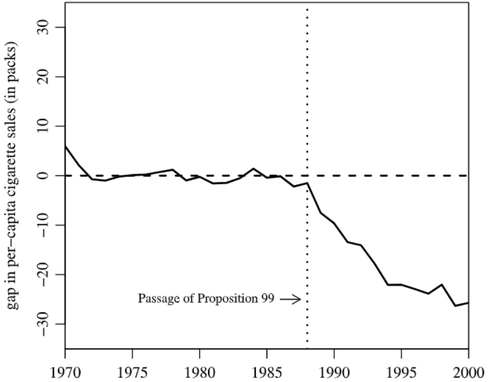


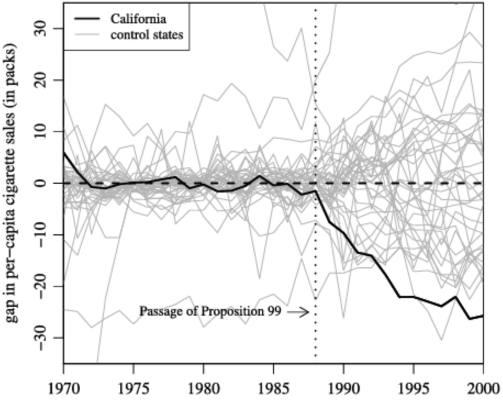
Prop. 99: Inference

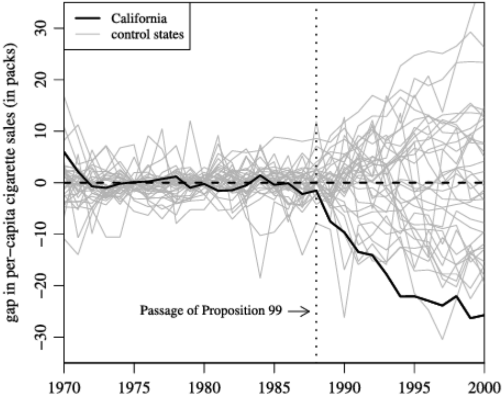
- Repeat analysis for California
- But with each donor pool state as "treated"
- One gray line for each placebo state
- If Prop. 99 has a real effect, what should the other gray lines look like?



Left panel has all states; right panel limits to 34 states with pre-treatment RMPSE $<$ 20 times CA's

Prop. 99: Possibly Stronger Inference





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Prop. 99: Alternative Inference

- Divide squared treatment-control difference after treatment by
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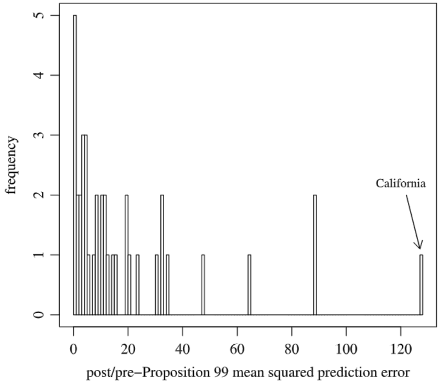
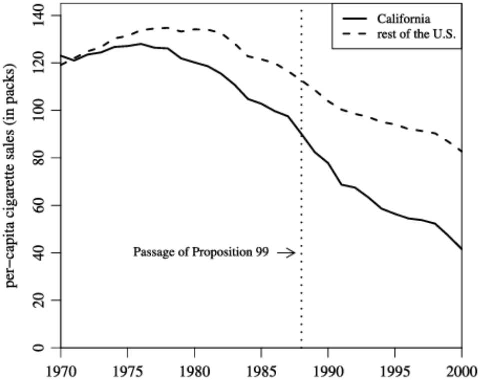


Prop. 99: Comparison

No synthetic control



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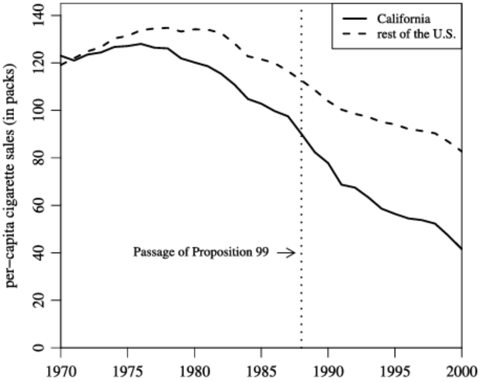
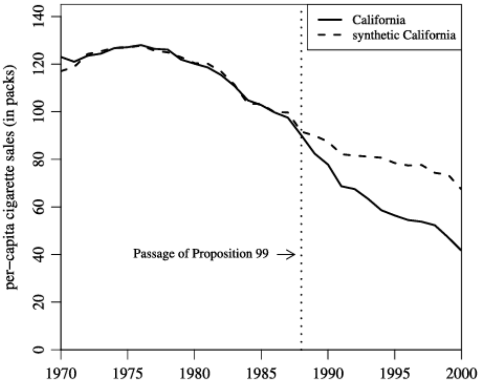


With synthetic control



Upcoming: The End is in Sight

- Lecture 12: In-class workshop and how to explain a causal strategy
- Lecture 13: Presentations
- Lecture 14: Presentations