# Tutorial 6: Functions 

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Mid-way through the semester, we take a break from data visualization to teach you how to create your own functions in R .
Most of the commands you use in R are functions themselves, so you already know how to use a function (even if you didn't realize it). Today you will learn how to make your very own $R$ function.
Make sure to write today's tutorial in an R script, rather than R Markdown. In addition, put
rm(list=1s())
at the beginning of your code. If strange things are happening in your code, consider running it from the beginning, and the above command will clear everything in R's memory. I tell you write a R script so that you can more easily keep track of what functions are defined where and apply to what.
As to when to use a function, in the immortal words of Hadley Wickham, "You should consider writing a function whenever you've copied and pasted a block of code more than twice (i.e. you now have three copies of the same code)" (see this here).

The ability to create functions is one of the most powerful tools that statistical programming gives you. Good programming relies heavily on functions. Cut and paste is prone to errors. Furthermore, while cut and paste always seems easiest when you need to do things once, my experience with programming suggests that your first graph is never your last graph. When you need to make modifications, particularly ones that are the same across a set of graphs, you will be happy you chose to make a function.

This class gives you an initial introduction to functions. If you'd like to deeper, I recommend Hadley Wickman's Advanced R notes.

## A. What is a function?

In this section, we define what a function is and explain its parts.
The example below shows the bones of any basic function. The first part function. name is the name of the function. You can choose any name you'd like for the function.

```
function.name <- function(arg1, arg2){
    # stuff your function does
}
```

Beware, however, that if you use the name of an existing function in R, such as plot, when you call plot, you will get your new function, instead of R's usual 'plot'. Bottom line: stay away from existing names if at all possible. You can ask $R$ whether it has any functions of a given name by typing

```
? sum
## starting httpd help server ... done
```

When you type the above, a help window pops up - this means there is an existing function named sum. So don't use sum! What about dogs?

```
? dogs
```

\#\# No documentation for 'dogs' in specified packages and libraries:
\#\# you could try '??dogs'
$R$ tells us it can't find any functions called dogs, and you are good to go.
Every function also needs the word function - this is the same across all functions, and tells R that you are making a function. You cannot change this word.
The variables arg1 and arg2 are the inputs to the new function. They define what you can put into the function.

Once you've given R values for $\arg 1$ and $\arg 2$, R undertakes the commands inside the curly brackets. These commands are known as the "function body."

## B. A first function

## B.1. A very first function

Let's begin with a very simple function that takes one value to the power of the other. As we construct it, this function has arguments x and y . Whatever value you give R for x , it will take it to the power y . We call this new function summer.

Before I define it, I check to see if a function by this name already exists.

```
? summer
## No documentation for 'summer' in specified packages and libraries:
## you could try '??summer'
```

No function exists by this name. This check is not required, but it is good practice, since you can create trouble by naming your function with a name that already exists as a R function.
Now let's define the function:

```
summer <- function(x,y){
    x^y
}
```

In the summer function, the arguments are x and y . The body is $x^{y}$.
Having defined the function, we'd now like to call it. The most clear way of calling a function is to associate each argument with its value, as in

```
summer ( }\textrm{x}=1,\textrm{y}=2
```

```
## [1] 1
```

This call should look familiar. You've been using functions all semester. Now you're writing one of your own. The summer function returns a value of 1 . Note that

$$
1^{2}=1
$$

, so all is good. as it should.
Alternatively, you can make the same call by writing

```
summer (1,2)
```

\#\# [1] 1

This works, but is bad practice. Code like this is hard to decipher and debug.
Now try

```
summer(x = 2, y = 1)
```

\#\# [1] 2
Note that this does not yield the same outcome. Homework question 1 asks you why.
Finally, the call below does not work at all. We've specified nothing for y , and all arguments are mandatory unless a default value is specified (which we'll learn how to do in a bit).

```
summer(x = 1,)
```


## B.2. A Slightly More Complicated Function

The example in B.1. was so simple that you might wonder why we bother with functions. Let us start working toward something slightly more complicated.

Suppose you'd like to know the marginal tax rate for a specific income. Maybe you'd like to automatically print a chart title that says what the marginal tax rate of the mean income is, for example (so you can update the picture without looking up the marginl tax rate each time).

The marginal tax rate is the tax you pay on your last dollar of earnings. In the US system, rates are progressive, so that higher incomes pay higher tax rates. In other words, your first $\$ x$ of income is taxed at rate $a$. Income greater than $x$, but less that $y$ is taxed at rated $b$, where $b<a$. The rate associated with your "tax bracket" is your marginal tax rate.
A starting point for this kind of work is a function that delivers a marginal tax rate based on an input income. We do this below, with a thanks to Bankrate for helpful marginal tax rates (for single people; page also has married, if you're curious).

```
single.marg.tax.rate <- function(income){
    mr <-
        ifelse(income < 9325,0.10,
            ifelse(income < 37950, 0.15,
                ifelse(income < 91500, 0.25,
                ifelse(income < 191650, 0.28,
                    ifelse(income < 416700, 0.33,
                        ifelse(income < 418400, 0.35, 0.396))))))
    print(pasteO("marginal tax rate for income ",income, " is ", mr))
}
```

This function takes the argument income and finds the bracket into which that income fits. It then outputs the tax rate and income in a print statement.

Give it a try!
Here's my first attempt:

```
single.marg.tax.rate(income = 10000)
## [1] "marginal tax rate for income 10000 is 0.15"
```

This seems to find the right marginal tax rate, according to the Bankrate page.
Does it work for other incomes?

```
single.marg.tax.rate(income = 50000)
## [1] "marginal tax rate for income 50000 is 0.25"
single.marg.tax.rate(income = 500000)
## [1] "marginal tax rate for income 5e+05 is 0.396"
```

You could do what we just did by copying and pasting the ifelse() statement a number of times, or by looking up values by hand. So why bother with this function? The function really shines when you've made a mistake with one number in the brackets. If you make a mistake and you've done things by hand, you need to check every individual decision you made. If you made a function, you re-code the function and you are done.

## B.3. A function in a separate file

Sometimes you build a function that you would like to use in multiple programs. If you'd like to do this, you can put your $R$ function in its own . $R$ file.

For example, I put a variant of the summer function we created above in a separate new R file, and saved it as summer2_func.R (don't use dots in the file name, except for the .R extension). My file looks like

```
summer2 <- function(x,y,z){
    x`y + z
}
```

I can now call this function and get a result:

```
source("H:/pppa_data_viz/2018/tutorials/lecture12/summer2_func.R")
summer2(x = 5, y = 3, z = 1)
## [1] 126
```


## C. Other Function Argument Basics

In this section, we discuss more features of function arguments: how they work, how you name them, and setting defaults.

## C.1. More on function arguments

Suppose that we would like to run the function summer2, but we don't want to add anything to $x^{y}$ (what the z variable does).
Let's try to run it two ways:
summer2 $(\mathrm{x}=5, \mathrm{y}=3, \mathrm{z}=0)$
\#\# [1] 125
This one runs, and properly gives us $5^{3}+0=45$.
Now let's try
summer2 $(\mathrm{x}=5, \mathrm{y}=3)$
This one should give an error message. Why? $R$ is trying to evaluation $x^{\wedge} y+z$, but can't find a value for $z-$ so it breaks.

To make sure you understand why R is breaking, look at the following example:

```
summer3 <- function(x,y,z){
    x^y
}
summer3(x = 5, y = 3)
## [1] 125
```

This does not generate an error. The homework asks you why, even without a value for $\mathbf{z}$.
Note that you can also put R objects into a function call. Let's let natl.mn.income be $\$ 53,719$, which is the 2014 US mean income. We'll then use this object in the summer function call.

```
natl.mn.inc <- }5371
summer(x = natl.mn.inc,y = 1)
## [1] 53719
```


## C.2. Defaults

One way to avoid the error we have in section C.1. from calling summer2 $(5,3)$ would be to set a default value for $z$. Let us set the default value for $z$ as 0 .

```
summer4 <- function(x, y, z = 0){
    x`y + z
}
summer4(x = 5, y = 3)
## [1] 125
```

Now if you don't specify a value for $\mathbf{z}, \mathrm{R}$ assumes that it is zero. If you do specify a value, that value replaces zero.

## C.3. Calling the right type of variable

You should also be careful that the type of input argument you give to the function matches how the function will use that argument.
The text below yields an error:

```
summer(x = "fred", y = "ted")
```

Explain why in homework question 3.

## D. What the function outputs

Sometimes you'd like a function to just calculate something and print the result to the screen. Other times, it's helpful to have a function return something to you that you can use in the rest of the code. For example, suppose we'd like to use the marginal tax rate that the function single.marginal.tax.rate creates.

Can I work with this new marginal tax rate in the rest of the code?

```
single.marg.tax.rate(income = 500000)
```

taxes.paid <- (500000 - 418401) $* \mathrm{mr}$

This second command gives an eror. Why is this? Didn't we just create mr in this function? Why doesn't this object now exist?
This brings up a key element of functions. Everything that you create in the function is "local" to the function unless you specifically tell R you want to take it out of the function. To tell R to take something out of the function, you need to "return" the value. "Returning" the value means taking something that exists just in the function and making it exist in the rest of the code as well. We will learn how to do this in this section.

As an aside, when you write mr in plain R code, R will print the value of mr . When you write mr inside a function, $R$ doesn't print the value of $m r$ to the console. To see the value of $m r$, you need to write print (mr). I also illustrate this point in the code below.

Running our original function again, we see that just running it in plain code delivers the marginal tax rate to the console.

```
single.marg.tax.rate(income = 500000)
## [1] "marginal tax rate for income 5e+05 is 0.396"
```

Making the function deliver something to a new object prints nothing, but gives no error.

```
out <- single.marg.tax.rate(income = 500000)
## [1] "marginal tax rate for income 5e+05 is 0.396"
```

Let's look at the new object:
out
\#\# [1] "marginal tax rate for income $5 \mathrm{e}+05$ is 0.396 "
It's a text string! Which is the last thing the function did.
If we want the marginal tax rate as a number, we need to modify the function. Let's make a new function called single.marg.tax.rate.v2.

```
single.marg.tax.rate.v2 <- function(income){
    mr <-
        ifelse(income < 9325,0.10,
            ifelse(income < 37950, 0.15,
```

```
        ifelse(income < 91500, 0.25,
        ifelse(income < 191650, 0.28,
            ifelse(income < 416700, 0.33,
                        ifelse(income < 418400, 0.35, 0.396))))))
    print(paste0("marginal tax rate for income ",income, " is ", mr))
    mr
}
```

Let's now run the function, and also put its output into out2.

```
single.marg.tax.rate.v2(income = 500000)
## [1] "marginal tax rate for income 5e+05 is 0.396"
## [1] 0.396
out2 <- single.marg.tax.rate.v2(income = 500000)
## [1] "marginal tax rate for income 5e+05 is 0.396"
out2
## [1] 0.396
```

This output is now a number, since listing mr is the last thing the function did. We can now use out2 in our main program. Below I calculate the taxes paid, for someone earning $\$ 500,000$, on income above $\$ 418,401$. (This person does not pay the marginal tax rate of 39.6 on all of their income - just income above $\$ 418,401$. For income between $\$ 416,701$ and $\$ 418,401$, the person pays a rate of 35 percent. Below that - from about $\$ 200,000$ to $\$ 400,000$ they pay 33 percent.)

```
taxes.paid <- (500000 - 418401)*out2
```

taxes.paid
\#\# [1] 32313.2

## E. Functions for doing things with dataframes

To illustrate the value of functions, let's automate some repetitive operations. A good practice when building a function is to write out one instance of what you'd like to do in plain code. Then work on the function that automates it. You can write the function directly, but this is best for when you are quite comfortable with functions.

## E.1. Load DC crash data

Begin by loading the csv with crashes in DC , found here.
You can do this in a bunch of different ways. Here are two alternatives. The first is to download the spreadsheet, read it into R, and find out what variables this dataframe has. I've done this already, and to be sure you're using the same data, you can use the data I downloaded and find it here.

Here is the code to read in the csv file and find the names of the variables.

```
crashes <- read.csv("H:/pppa_data_viz/2021/tutorial_data/tutorial_06/20210221_Crashes_in_DC.csv")
```

The second way to grab the data is by using the API. Grab the GeoJSON format data and get rid of the spatial component.

```
# load the required library
require(geojsonsf)
```

```
## Loading required package: geojsonsf
require(tidyverse)
## Loading required package: tidyverse
## -- Attaching packages -------------------------------------------------
## v ggplot2 3.3.2 v purrr 0.3.4
## v tibble 3.0.4 v dplyr 1.0.2
## v tidyr 1.1.2 v stringr 1.4.0
## v readr 1.4.0 v forcats 0.5.0
## -- Conflicts ---------------------------------------------
## x dplyr::filter() masks stats::filter()
## x dplyr::lag() masks stats::lag()
require(sf)
## Loading required package: sf
## Linking to GEOS 3.8.0, GDAL 3.0.4, PROJ 6.3.1
# name the location of the data
crashessjson <- "https://opendata.arcgis.com/datasets/70392a096a8e431381f1f692aaa06afd_24.geojson"
# load the geojson as a sf
crashessf <- geojson_sf(crashessjson)
# make a copy of the dataframe
crashesnosf <- crashessf
# get rid of the geometry
st_geometry(crashesnosf) <- NULL
```

I am using a file that was last updated August 12, 2020.

## E.2. Understand the data

Before we get started on iterative programming, let's first take a look at this dataframe to understand how it's set up and what variables it has.

```
str(crashes)
## 'data.frame': 244418 obs. of 60 variables:
## $ i..X : num -77 -77 -77 -76.9 -76.9 ...
## $ Y : num 38.9 38.9 38.9 38.9 38.9 ...
## $ OBJECTID : int 190508290 190508291190508292 190508293 190508294 190508295 1905
## $ CRIMEID : int 28645003 28645013 28644550 28644553 28644582 28644608 28644614 2
## $ CCN : chr "21002328" "21002815" "21002323" "21002335" ...
## $ REPORTDATE : chr "2021/01/07 03:52:24+00" "2021/01/07 04:11:09+00" "2021/01/06 06
## $ ROUTEID : chr "0" "0" "0" "0" ...
## $ MEASURE : num 0 0 0 0 0 0 0 0 0 0 ...
## $ OFFSET : num 0.68 4.19 2.89 3.53 5.3 ...
## $ STREETSEGID : int NA NA NA NA NA NA NA NA NA NA ...
## $ ROADWAYSEGID : int NA NA NA NA NA NA NA NA NA NA ...
## $ FROMDATE : chr "2021/01/05 05:00:00+00" "2021/01/06 05:00:00+00" "2021/01/05 05
## $ TODATE : chr "1970/01/01 00:00:00+00" "1970/01/01 00:00:00+00" "1970/01/01 00
## $ MARID : int 26775 46921 313187 289486 9414 239363 296206 228314 330694 24455
## $ ADDRESS : chr "411 FRANKLIN STREET NE" "2452 ALABAMA AVENUE SE" "NAYLOR ROAD S
## $ LATITUDE : num 38.9 38.9 38.9 38.9 38.9 ...
## $ LONGITUDE : num -77 -77 -77 -76.9 -76.9 ...
```



## E.3. Create new output via a function with one input

Suppose we'd like to run a couple of commands on multiple variables. This is something we can do with a function. Let's suppose that we'd like to * output summary statistics using summary () * look at number of missings with table() * look at distribution of outcomes with table()

Here's an example of this code for the variable TOTAL_VEHICLES
summary (crashes\$TOTAL_VEHICLES)
\#\# Min. 1st Qu. Median Mean 3rd Qu. Max.

```
## 0.000 
table(crashes$TOTAL_VEHICLES)
##
\begin{tabular}{lrrrrrrrrrr} 
\#\# & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\#\# & 1279 & 30871 & 192284 & 15779 & 3111 & 734 & 215 & 92 & 22 & 18
\end{tabular}
## 
table(is.na(crashes$TOTAL_VEHICLES))
##
## FALSE
## 244418
```

Now suppose we'd like to do this for all the variable that start with "TOTAL." We'll make a function to do this. But first let's start with a function that does not work. I do this to point out how you need to adjust your coding to make a function work.

You can try to run this, and it should generate an error:

```
sumup <- function(varin){
    print(summary(crashes$varin))
    print(table(crashes$varin))
    print(table(is.na(crashes$varin)))
}
sumup(varin = TOTAL_VEHICLES)
## Length Class Mode
## 0 NULL NULL
## < table of extent 0 >
## < table of extent 0 >
```

When you write crashes\$varin, R looks for a variable named varin - it doesn't replace varin with the text you're passing in.
Instead, you need to write the variables inside a double bracket [[]], rather than the dollar sign notation. Here R does know to replace the varin marker with its value.

```
sumup2 <- function(varin){
    print(paste0("inside the function for variable ",varin))
    print(summary(crashes[[varin]]))
    print(table(crashes[[varin]]))
    print(table(is.na(crashes[[varin]])))
}
sumup2(varin = "TOTAL_VEHICLES")
## [1] "inside the function for variable TOTAL_VEHICLES"
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.000 
##
## 
### 
##
## FALSE
## 244418
```

Now that we know the function works, we can use it for other variables:

```
sumup2(varin = "MAJORINJURIES_DRIVER")
## [1] "inside the function for variable MAJORINJURIES_DRIVER"
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.00000 0.00000 0.00000 0.07069 0.00000 7.00000
##
## 
```



```
##
## FALSE
## 244418
sumup2(varin = "MINORINJURIES_DRIVER")
## [1] "inside the function for variable MINORINJURIES_DRIVER"
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.000 0.000 0.000 0.173 0.000 11.000
##
## 
## 207036 
##
## FALSE
## 244418
sumup2(varin = "TOTAL_BICYCLES")
## [1] "inside the function for variable TOTAL_BICYCLES"
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.00000 0.00000 0.00000 0.01775 0.00000 2.00000
##
## 0
## 240122 4254 42
##
## FALSE
## 244418
sumup2(varin = "TOTAL_PEDESTRIANS")
## [1] "inside the function for variable TOTAL_PEDESTRIANS"
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.00000 0.00000 0.00000 0.05604 0.00000 12.00000
##
## 
##
## FALSE
## 244418
```

Now we've generated summary statistics for a set of variables. If we wanted to add another summary statistic, we could just modify the function.

## E.4. Create new output via a function with one input

But what we've just done doesn't really illustrate the power of functions because we are just changing one element each time we're going through the function. If you're familiar with loops in other programming, this is doing what a loop does.

A function can iterate through two (or more) conditions, which makes it more powerful than a loop. Let's use that power to look at the summary statistics we already have, but for day and night separately.
Unfortunately, this dataframe doesn't have day and night - but it does have a a time from which we can create a day and night variable

```
crashes$time <- substr(x = crashes$REPORTDATE, start = 12, stop = 13)
table(crashes$time)
\begin{tabular}{lrrrrrrrrrrr} 
\#\# & & & & & & & & \\
\#\# & 00 & 01 & 02 & 03 & 04 & 05 & 06 & 07 & 08 & 09 & 10 \\
\#\# & 9549 & 8594 & 9121 & 7558 & 6354 & 101851 & 3813 & 3408 & 3306 & 3396 & 3069 \\
\#\# & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\
\#\# & 3094 & 3684 & 5210 & 6730 & 7262 & 6999 & 7418 & 7701 & 6974 & 5996 & 7040 \\
\#\# & 22 & 23 & & & & & & & & & \\
\#\# & 7959 & 8332 & & & & & & & & & \\
\#2021/01/07 \(03: 52: 24+00\) & & & & & & & & \\
\#1234567890123456789012
\end{tabular}
```

I look at the distribution of times here and I'm a little suspicious - does the evening shift end at 5 am? I don't believe that most accidents occur at 5 am . But for the purposes of this assignment, we'll just continue and create a variable that is day or night.

```
crashes$day <- ifelse(test = as.numeric(crashes$time) >= 6 & as.numeric(crashes$time) <= 18,
    yes = 1,
    no = 0)
table(crashes$day)
##
## 0 1
## 179328 65090
```

Now we have a variable that is equal to 1 if the accident took place during the day and 0 if the accident took place at night.

Let's use this new variable to re-assess our previous analysis. Now our function will make data subsets for day and or night and re-do the same analysis.

```
sumup3 <- function(varin, daytime){
    c2 <- crashes[which(crashes$day == 1),]
    print(paste0("inside the function for variable ",varin, " when day == ",daytime))
    print(summary(c2[[varin]]))
    print(table(c2[[varin]]))
    print(table(is.na(c2[[varin]])))
}
sumup3(varin = "TOTAL_VEHICLES", daytime = 1)
## [1] "inside the function for variable TOTAL_VEHICLES when day == 1"
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.000 2.000 2.000 1.959 2.000 16.000
##
\begin{tabular}{lrrrrrrrrrrrr} 
\#\# & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
\#\# & 255 & 9541 & 49774 & 4241 & 918 & 243 & 69 & 33 & 5 & 6 & 1 & 1
\end{tabular}
## 13 16
## 1 1
##
## FALSE
```

This general principle of referring to variables in double brackets works in all Base R commands, and the general principles of functions work for all $R$ commands.
However, there are some strange nuances with tidyverse commands, as we'll explore in the next section.

## F. Functions and tidyverse

In this step, we explain why it is that tidyverse commands are tricky, and then we turn to how to deal with this, and then how to make functions for graphs.

## F.1. Why tidyverse is Tricky

Now we turn to how to use functions with tidyverse commands. One of the things that makes tidyverse commands pleasant to use relative to Base R is the ability to dispense with the full dataframe name for variables.

For example, compare Base R's way of subsetting with tidyverse's:

```
# base R
dayonly <- crashes[which(crashes$day == 1),]
# tidyverse
dayonly <- filter(.data = crashes, day == 1)
```

You probably find the filter way easier to understand. It is, but it is not easier to put in a function.
Here's why. It seems that this kind of function, where we tell R some input variables, should work.

```
graphit <- function(xvar,namer1){
    ggplot() +
        geom_histogram(data = crashes,
            mapping = aes(x= xvar)) +
        labs(title = paste0("Histogram of ",namer1),
            x = namer1)
}
```

But when you test it, it does not:

```
graphit(xvar = WARD,
    namer1 = "Ward")
```

This is because of the non-standard way in which all tidyverse packages evaluate R code. For more on that, see this document.

## F.2. Fixes

However, there is a happy fix (many thanks to this post). Put the variables you want to refer to inside double curly braces: $\{\}\}$. Here is an example. Notice that when we call the function, the variable name is not in quotes, as it was above. Here it is unquoted, as it generally is in tidyverse commands.

```
graphit2 <- function(xvar,namer1){
    ggplot() +
        geom_histogram(data = crashes,
            mapping = aes(x = {{xvar}})) +
        labs(title = pasteO("Histogram of ",namer1),
            x = namer1)
```

graphit2(xvar = TOTAL_VEHICLES, namer1 = "total vehicles involved in crash")
\#\# `stat_bin()` using `bins \(=30 `\). Pick better value with `binwidth`.

Histogram of total vehicles involved in crash


You should now have enough basics to be able to write functions on your own. I strongly encourage you to practice using them. They streamline both code and thinking, and they are the key to programming efficiently and reliably.

## G. Homework

1. In section B.1., why do
```
summer(x = 1, y = 2)
summer(x = 2, y = 1)
```

not yield the same output? Write in math what each one does.
2. In section C.1., why does the call $\operatorname{summer3(x=5,y=3)\text {returnavaluewhen}\operatorname {summer}2(x=5,y=}$ 3) does not?
3. In C.2., why does summer ( $\mathrm{x}=$ "fred", $\mathrm{y}=$ "ted") yield an error?
4. Fix the function in part E to remove the graph background.
5. Make a function that automates a graphics operation of interest to you, using a dataset not from this tutorial.

