Why We Still Don't Know More About Housing Supply

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Abstract

This paper develops implications from models of urban spatial structure for estimates of the long-run supply of housing across cities. It demonstrates that housing supply elasticity varies inversely with city size and transportation cost, and directly with cost of structure inputs and rural land in both classical and neoclassical models. Topographic features and planning regulations that reduce the share of land available for development do not influence supply elasticity if they apply uniformly. Finally, elasticity depends on the location within the city where housing prices are measured. These relations have the potential to confound empirical estimates of the determinants of housing supply elasticity, but are less problematic for numerical simulation models.

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I. Introduction

What is the long-run own price elasticity of housing supply and why does it vary across cities? These questions are important because factors that lower supply elasticity in highly productive cities could have substantial welfare effects identified by Hsieh and Moretti (2019), among others. However, estimating housing supply functions is difficult. The title to this paper is a reference to DiPasquale (1999), which focused attention on several empirical challenges. The results presented here, formally derived from urban spatial models, are consistent with her conclusions. Until these issues are adequately addressed, an understanding of the determinants of supply elasticity will remain elusive.

The empirical literature is divided into studies of short-run and long-run elasticities. Short-run estimates have a putty-clay assumption, with supply responses measured by new construction on vacant land over a period of one decade or less. Long-run elasticity involves putty-putty adjustments over perhaps 40 years. Long-run supply responses include teardowns, rebuilds, and renovations, as well as responses to changes in infrastructure, including transportation facilities.

Given the difficulty of measuring changes in housing services or even interior space, the empirical literature has generally measured changes in the number of housing units. Measuring units rather than aggregate space is consequential because Sarkar (2011) reports that, over the 1970-2000 period covered in empirical work discussed below, the average new unit increased in size by 20% and the fraction with garages rose from 59% to 77%. Existing units were also substantially upgraded so that the change in number of units over time does not reflect changes in housing services supplied over the period. Theory allows the derivation of results for short-run and long-run price elasticities of both housing space and units holding constant preferences for housing and household size that cause unit sizes to vary over time.

Due presumably to a lack of reliable panel data on residential rents, the empirical literature has concentrated on variation in asset prices. This is problematic given that Carrillo, Harris, and Yezer, (2023) show that the standard deviation in asset prices across cities is twice the standard deviation in rental prices. However, the theoretical analysis can be performed using changes in rental prices that reflect the use value of housing rather than confusing use and option value by relying on changes in asset prices. Lastly, theory allows the effect of variation in measures of housing price to be compared. It is possible to measure price change at a fixed location, the CBD, the mean distance from the CBD, or the difference between prices of existing and newly constructed units.

Two theoretical approaches to modeling the urban housing market are considered here. First is a classical model in which there is one identical house per unit land and homogenous households consume that single unit whose price varies with commuting cost. The classical model is important because it has been used in Saiz (2010) and Green et al. (2005) to justify empirical testing.

Second is a neoclassical model in which housing is produced using land and structure inputs that are substitutable and households maximize utility subject to a price per unit housing space and commuting cost, or equivalently wages, that vary with distance to the city center. Cosman et al. (2018) also use a neoclassical model to develop implications for short-run supply elasticity. Although their model of housing supply in cities is quite different, several of the arguments made here are logically consistent with it.[1]

The next section of this paper reviews literature on the long-run elasticity of housing supply in cities. In addition to direct estimate of supply functions, implied elasticities obtained from numerical simulation models and estimates of the urban wage premium are considered. This literature review highlights the substantial variation in the estimates of supply elasticity associated with differences in method. Next, there is a brief discussion of literature on short-run supply elasticity which has some relation to the theoretical results generated here.

The heart of the paper is a formal derivation of propositions regarding the theoretical determinants of housing supply in a classical model and a complementary analysis in a neoclassical model. The neoclassical model includes the extra complication that short and long-run supply elasticities differ because, in the long run, the current stock of built housing may be redeveloped at an alternative density, i.e. long-run supply is putty-putty whereas short-run is putty-clay. The paper concludes with implications of theory for the possibility of successfully estimating long-run housing supply elasticities and implications for use of alternative approaches to estimating supply elasticity.

II. Long-run Supply Elasticity Estimates

This section concentrates on a review of estimates of the long-run elasticity of housing supply considering time periods in which clearance and construction at an alternative density is possible as well as provision of infrastructure to accommodate growth. One approach is to estimate

[1] Their model requires housing to be supplied on previously undeveloped land whereas the long run model here assumes that land used for housing can be redeveloped at a higher density and specific functional forms for the housing production function and housing price gradient are not assumed.

housing supply equations directly by relating changes in the number of housing units to changes various measures of the change mean or median price of housing units. Another method relies on neoclassical models of the urban housing market that allow numerical simulation of the relation between changes in various measures of the rental price of housing space and the quantity of either housing units or interior space. Finally an implied supply elasticity can be computed using estimates of the urban wage premium. The results of all three of these approaches are presented briefly here.

In terms of citations, the two most prominent estimates of the long-run price elasticity of housing units in U.S. cities are Green et al. (2005) and Saiz (2010). The former paper first estimates supply elasticities for 45 U.S. cities over 18 years, 1979 to 1996. The range of elasticities for the full sample of 45 cities is 29.9 to −0.30 with a median of 5.2 for San Diego.[2] A second stage regression relates these estimates to possible determinants of differences in elasticity and finds little agreement with theory in that elasticity varies directly with city population and the effect of commuting cost is non-significant. It does report a negative and significant relation between elasticity and a regulatory index.[3]

Saiz (2010) estimates housing supply elasticities for 95 U.S. cities based on the change over 1970 to 2000 in median city housing price and the number of housing units. The estimates range from 5.45 in Wichita to 0.60 for Miami with a median of 1.61 in Washington, Providence, and Phoenix. The factors considered to influence elasticity are measures of buildable land and

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[2] Considering only estimates significantly different that zero, reduces the inference to 22 cities with a range of 21.6 to 1.43 and a median of 5.3 for Birmingham.

[3] The authors report that theory suggests measuring housing rental price change at a given point but data availability causes them to use median asset price changes.

regulation which both reduce availability. This paper uses a classical land market model with fixed proportions of housing and land to establish the relation between.

While the negative effect of regulation on supply elasticity is consistent across Green et al. (2005) and Saiz (2010) other aspects of the estimates diverge. For example, the median elasticity of the former is almost five times the latter and the ranges of estimates vary by a similar proportion. To compare the implications of the difference in these elasticity estimates consider that, if the number of housing units increases from 1 to 10 million, and supply elasticity is 5.2, then house price would increase by 173% compared to a 565% if supply elasticity is 1.61. Green et al. (2005) find transportation cost, measured as congestion, does not lower elasticity as theory leads them to expect and Saiz (2010) ignores the variable. Clearly transportation cost should have a substantial effect on supply elasticity because as commuting cost approaches zero the supply of equally attractive urban land available for housing becomes perfectly elastic and housing supply elasticity follows accordingly. There are econometric issues associated with identification of long-run housing supply relations, particularly the complex relation between real estate and transportation in cities. Some of these issues have been discussed by Davidoff (2016) and are independent on the theoretical points raised in this paper.

An alternative to direct empirical estimation is to infer long-run elasticity of housing supply from a numerical urban simulation model. Larson, et al. (2022) find that, for a model calibrated to Chicago, the supply elasticity of housing units with respect to changes in the median price of a housing unit is 6.25. The simulation distinguishes supply of units from supply of interior space. The elasticity of unit size with respect to units supplied is 0.91 so that the elasticity of interior space with respect to price is 6.25 (0.91) = 5.7. The simulation also demonstrates that the elasticity of housing supply is very sensitive to the location at which the housing price change is measured.

Supply elasticity with respect to housing price change at the city center is only 3.1. Furthermore, the elasticity of supply does not vary with the fraction of land available for housing, assuming that fraction is not a function of distance, and it does not depend on planning regulations limiting density it they are uniformly applied.[4] The theoretical results in this paper demonstrate why the findings in this type of numerical simulation model should be expected.

A third source of estimates of the long-run supply elasticity of housing is to use inference based on the urban wage premium, the elasticity of wage rates with respect to city population. Chauvin, et al (2017) estimate this elasticity using alternative specifications and report results ranging from 0.053 to 0.076. Liu (2017) disaggregates estimates of the elasticity of wages with respect to college major and reports varying results from 0.033 for engineering to 0.064 for economics and business. Following the literature on converting housing supply space elasticity into an implied wage differential as in Hsieh and Moretti (2019), the implied housing supply is computed from a given urban wage premium is the quotient of the urban wage premium and the share of housing in consumption. If the share of rent in income is 0.30 and the urban wage premium based on the estimates reported above is 0.06 then the implied elasticity of housing space supply for U.S. cities is approximately 5.0. This elasticity should be compared to the relation between housing rental price and housing space rather that that between housing units and asset price. Adjusting for the elasticity of unit size with respect to housing price, the elasticity of supply of housing units would be 5.0/0.91 = 5.5.

[4] The results show that effects of density limits on housing supply elasticity depend entirely on how binding they are and where within the city they are binding. For example, if density limits only bind near the city center, they may raise housing supply elasticity measured using median price change because new housing is built in suburban areas where prices are lower. Binding zoning regulation changes the location of the average housing unit compared to laissez faire.

Summarizing the alternative approaches to estimating the long-run elasticity of supply of housing reviewed above, there is remarkable agreement between the results of the numerical urban simulation model and estimates of the urban wage premium. The elasticity of supply of housing units with respect to rental price is between 5 and 6. This is also close to the average supply elasticity reported in Green et al. (2005), but the range across cities that they report does not seem credible. Estimates in Saiz (2010) stand out as being substantially below the other values. Furthermore, the general failure to consider effects of transportation cost in the econometric estimates is a shortcoming of this literature.

III. Short-run Supply Elasticity Estimates

In addition to long-run housing supply elasticity, there is an active literature on short-run supply elasticity in which new supply is provided by construction of new units on sites not currently used for occupied units. The theory developed here considers such putty-clay models of housing supply but that is not the primary focus of this paper. For example, Goodman (2013) has demonstrated substantial non-linearity in the empirical housing supply schedule of cities which implies that supply is hysteretic. This view has been confirmed recently by Aastveit and Anundsen (2022) who find that measured supply elasticity depends on the size and sign of the shock to each city. This paper abstracts from problems of local urban amenity shocks and urban decline.

Recently Baum-Snow and Han (2022) have demonstrated that the short-run elasticity of housing supply varies across city neighborhoods. The authors also provide an excellent review of the literature on housing supply estimation and its policy implications.

Short-run increase in housing supply are the result of building on formerly vacant land or teardowns of existing structure. However, real option models of the decision to develop, extending from Capozza and Heltzsley (1990), demonstrates that the supply of vacant land and the timing of teardowns depends on expectations of future price changes. Put another way, both recent price changes and future price expectations taken together determine the short-run supply response of a housing market, complicating empirical estimates.

IV. Housing Supply Elasticity in a Classical City

As noted above, there are two approaches to modeling the price elasticity of housing supply. The long-run assumption of putty-putty construction assumes that, in response to a change in price, the entire housing stock is subject to modification. Additional housing space may be built on land currently occupied by existing units or on undeveloped land. In contrast a short-run assumption of putty-clay housing only allows construction on formerly vacant land, presumably converted from non-urban uses at the city edge. In the case of a classical city, the Leontief production function fixes the housing to land ratio regardless of housing price. Accordingly, the only possibility for adding housing space or units is conversion of land at the edge of the city. Therefore, the results of the classical model presented here hold for both long and short-run supply responses to changes in the price of housing. As noted above, the classical model has been used by Saiz (2010) and Green et al. (2005) to justify previous empirical estimates of housing supply.

The distinguishing characteristic of a classical city is that households consume a standard quantity of housing, h, and the housing production function is Leontief, so that:

$$H = Min\ [\alpha l, \beta s] \qquad (IV\text{-}1)$$

where H is the quantity of housing space, l is land, s is structure inputs, and α and β are parameters reflecting output per unit input. Producers of housing will set $\alpha l = \beta s$ so that housing output is a simple multiple of land, $H = \alpha l$. Classical households consume a fixed amount of housing, h, which is normalized to unity to economize on notation. The housing producer's cost function at any location is:

$$C = rl + is \qquad (IV\text{-}2)$$

where r is the rental price of land at that location and i is the rental price of structure inputs which is assumed invariant. If competition forces developers to set price equal to average cost then:

$$C/H = rl/H + is/H = rl/\alpha l + is/\beta s = r/a + i/b = p \qquad (IV\text{-}3)$$

with p equal to the rental price of a housing unit and H is both the number of units and the amount of space supplied because h was normalized to unity.

Households either must commute to the city center or earn wages that are lower than center city wages by the amount of transportation costs to the center. Letting p_o be price of housing units at the city center, and k indicate distance to that center, the rental prices of housing and land at distance k are given by:

$$p_o = r_o/a + i,/b; \;\; p_k = p_o - tk, \;\; and \; r_k = a(p_k - i/b) = a[p_o - tk - i/b] \qquad (IV\text{-}4)$$

Differentiation of *(IV-4)* with respect to k yields $dp/dk = -t$ which is a classical version of Muth's equation under the assumption that housing consumption is constant and equal to unity and transportation cost per unit distance is constant.

If the city limit is at k^*, p^* is housing price at that limit and the opportunity cost of urban land, commonly called the rental price of agricultural land, is r_A, then:

$$r_A = r^* = a[p^* - i/b] = a[p_o - tk^* - i/b] \;\; or \; k^* = (p_o - p^*)/t \qquad (IV\text{-}5)$$

Housing supply, both space and units, is proportional to land in the city used for housing. Letting Λ equal the fraction of land available for residential construction, the total supply of both city housing, H, and number of units, N, is given by:

$$H = \int_o^{k^*} (\Lambda 2\pi\alpha)\, k\, dk = (\Lambda\pi\alpha)\, k^{*2} \qquad \text{(IV-6)}$$

While the assumptions of the classical model may appear to depart from reality, thus far the empirical literature on housing supply elasticity has appealed to classical models to support stochastic specification of equations used to estimate supply elasticity.[5]

The influence of planning or topography on housing supply elasticity is reflected in two parameters of the classical model. First limits on building density, if binding, tend to lower α by requiring more land per structure than would be required given the production technology. Similarly, topography may limit construction density and lower α. Second Λ may be reduced by planning regulations that require open space or otherwise limit the fraction of land available for residences or by constraints imposed by topography on buildable land. Therefore, analyzing the effects of planning restrictions or topography on the elasticity of housing supply, requires that the role of these parameters in this classical model of housing supply elasticity be determined.

Proposition I: In a classical urban land market, the elasticity of supply of housing units and space in both the short and long run varies inversely with city population and transportation cost and directly with cost of agricultural land and structure inputs. Supply elasticity is not determined by the parameters Λ and α which represent factors such as topography and building regulations that limit the fraction of land available for housing or the density of housing units.

[5] For example see Saiz (2010) and Green, Malpezzi, and Mayo (2005) who appeal to versions of the classical model.

The starting point in exploration of the elasticity of housing supply is to define the term. In the classical model all housing units are the same size and the price per square foot is directly proportional to the price per housing unit. The notation E will indicate elasticity based on housing units and ϵ the elasticity based on housing space. In the classical model considered here, $E = \epsilon$. The percentage change in rental price of housing services may depend on location.[6] Initially consider price at the city center, p_o. This definition is appealing because rents at the city center do not require a measure of commuting cost. Accordingly, the definition of housing supply at the city center under the classical model is $\epsilon = dlog\ H/dlog\ r_o$.

For a circular city with proportion Λ of land available for housing, this is easily expressed by taking the logarithm of *(IV-6)*:

$$log\ H = log\ \Lambda + log\ \pi + log\ \alpha + 2\ log\ k^* \qquad (IV\text{-}7)$$

Where k is distance from the city center and k^* is the city limit. Given that households are mobile, there is an iso-utility condition that requires the rental price of housing units decline with increasing distance according to *(IV-5)*.

The elasticity of the city limit with respect to center city price is found using *(IV-5)*:

$$\partial log\ k^*/\partial log\ p_o = (\partial k^*/\partial p_o)\ (p_o/k^*) \ \ or$$

$$= (p_o/t)/\ [p_o\ /(p_o - p^*)/t] = p_o\ /(p_o - p^*)\ > 0 \qquad (IV\text{-}8)$$

This implies that the elasticity of city radius with respect to central rent is not constant and is only a function of central rent and $p^* = r_A/a + i/b$. Specifically, the change in elasticity of the city

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[6] The annual cost of housing, which determines demand, is based on rental price. However, supply is based on the asset price, which requires knowledge of the capitalization (cap) rate. In the discussion here, variation in the cap rate both within and across cities will be ignored. However, this is a very consequential issue for empirical estimation of housing supply elasticity.

radius with respect to rent, $d^2 log\ k^*/dlog\ p_o^2 = -p^*/(p_o - p^*)^2 < 0$. While the positive sign of the first derivative of $log\ k$ with respect to $log\ p_o$ is not surprising both the fact that the size of the derivative varies with p_o and p^* and the negative sign of the second derivative are less intuitive.

Continuing the focus on the elasticity of housing supply defined as $\epsilon_{po} = dlog\ H/dlog\ p_o$, totally differentiating *(IV-7)* yields:

$$dlog\ H = dlog\ \Lambda + dlog\ \pi + dlog\ \alpha + 2\ dlog\ k^* \qquad (IV\text{-}9)$$

Therefore the elasticity of housing supply as a function of p_o can be written as:

$$\epsilon_{po} = dlog\ H/dlog\ p_o = (dlog\ \Lambda/dlog\ p_o) + (dlog\ \pi/dlog\ p_o)$$

$$+ (dlog\ \alpha/dlog\ p_o) + 2(dlog\ k^*/dlog\ p_o) \qquad (IV\text{-}10)$$

Clearly this reduces to:

$$\epsilon_{po} = dlog\ H/dlog\ p_o = (dlog\ k^*/dlog\ p_o) = 2[p^*/(p_o - p^*)^2] > 0 \qquad (IV\text{-}10')$$

First, given that p_o has no effect on the parameters Λ, π, or α in the classical model, this establishes the second part of *Proposition I.* Under a straightforward definition of elasticity of supply based on p_o, changes in Λ and α, the two parameters reflecting the effects of planning or topography have no effect on housing supply elasticity. Second, the process of city growth in a classical model necessarily involves an increase in p_o. This means, as stated in the second part of *Proposition I* that in classical model housing supply elasticity, defined as $\epsilon_{po} = dlog\ H/dlog\ p_o$, is not constant. Instead, it varies inversely with the level of p_o, just as $dlog\ k^*/dlog\ p_o$ varies with p_o. Holding p^* constant, elasticity falls as p_o rises with city size, i.e. $\partial\epsilon_{po}/\partial p_o < 0$. This is a natural result of the geometry of cities, the definition of elasticity based on p_o, and the response of construction to the excess of rent above that required for rural land conversion and construction.

Finally, the two parameters, r_A and i, that raise p^*, *(IV-5)* $p^* = r_A + i$, raise the elasticity of housing supply as seen by differentiating *(IV-10')* yielding $\partial\epsilon_{po} / \partial p^* = -(p_o + p^*)/(p^* - p_o)^3 > 0$. This counterintuitive result arises because, as p^* rises, holding p_o constant, city population falls and that smaller size results in higher price elasticity of supply.

The final element of *Proposition I* concerns the inverse effect of transportation cost, t, on housing supply elasticity. This can be seen by from *(IV-10')* where $\epsilon_{po} = dlog\, H/dlog\, p_o = 2[p^*/(p_o - p^*)^2]$ and noting from *(IV-5)* that $(p_o - p^*)$ varies directly with t.

The results comprising *Proposition I* contrast with the assumption in the empirical literature that the elasticity of housing supply is not a function of city size, that transportation cost, can be omitted from estimates of supply elasticity and that differences in A or α, due either to topography or land use planning cause differences in supply elasticity. The results here imply that no such relations exist provided that A and α are not a function of k.[7] There is a further counterintuitive implication of *(IV-10')*. To the extent that regulation or higher construction costs raise p^* by raising i or r_A, the elasticity of housing supply will rise. These results hold for the relation between the percentage change in rental price at the city center and the percentage change in the number of housing units or the amount of housing space because these two are identical in the classical model.

Thus far, the analysis has been conducted in terms of price at the CBD, p_o. This is potentially easily observed, particularly as a rental price, in empirical work. However, prices can be observed at alternative locations. The next proposition extends the analysis to elasticity measures using prices measured at distances ranging to the city edge. This second proposition

[7] In the case of many cities, A and α likely vary with k. However, the pattern of that variation is far from uniform and may be either increasing or decreasing as a function of k.

implies that supply elasticity varies inversely with distance. This means that finding some way to control the distance at which price change is measured empirically within cities over time or across cities appears to pose an additional challenge for empirical estimation of housing supply elasticity.

Proposition II: Housing supply elasticity in a classical land market model falls with the distance from the CBD at which the price change is measured.

An alternative definition of the elasticity of housing supply might choose rent changes at some other location, e.g. $p^{\#}$, where $0 < k^{\#} < k^{*}$. The effect of this change in location on the elasticity of supply is straightforward. Given that $p_o = p^{\#} + tk^{\#}$, and $dp_o/dp^{\#} = 1$, it follows that

$$dlog\ p_o/dlog\ p^{\#} = dp_o/dp^{\#}\ (p^{\#}/p_o) = p^{\#}/(p^{\#} + tk^{\#}) > 0 \text{ and } < 1 \qquad (IV\text{-}12)$$

and $\epsilon_{p\#} = dlog\ H/dlog\ p^{\#} = (dlog\ H/dlog\ p_o)(dlog\ p_o/dlog\ p^{\#})$

$$= 2[p^{*}/(p_o - p^{*})^2][p^{\#}/(p^{\#} + tk^{\#}/h)] < \epsilon_{po} \qquad (IV\text{-}13)$$

Hence $\epsilon_{|p\#} = dlog\ H/dlog\ p^{\#}$ is equal to the product of $dlog\ H/dlog\ p_o$ and a term, $0 < [p^{\#}/(p^{\#} + tk^{\#}/h)] < 1$, which varies inversely with distance, $k^{\#}$. Housing supply elasticity measured by rents at a distance $0 < k^{\#} < k^{*}$ is uniformly smaller than supply elasticity measured by p_o and it decreases monotonically with distance $k^{\#}$. Thus, all problems with defining and measuring the elasticity of housing supply in cities that occur when constraining rental price to the city center also arise when price is measured at any other fixed radius between the center and edge. Furthermore, changing the location at which price is measured, changes the elasticity of housing supply.

For empirical purposes it might be attractive to use the average rent of all units in the city. The average location of a housing unit, noted $k^{@}$, is determined by:

$$k^{@} = [\int_o^{k^*} (\Lambda 2\pi/\alpha)\ k\ k\ dk\ ]/[\int_o^{k^*} (\Lambda 2\pi/\alpha)\ k\ dk] = (2/3)k^{*} \qquad (IV\text{-}14)$$

Unfortunately relying on mean rent does not solve the problems with rent at other locations. Setting $k^{\#} = (2/3)k^{*}$ does not remove any of the issues associated with a fixed location rent at $k^{\#}$.

Taken together, these considerations make the relation between percentage change in housing units or services and percentage change in the average housing unit rent, even if this could be measured empirically, truly problematic as a measure of the elasticity of housing supply in a classical urban land market model.

Note that, because both long-run and short-run supply responses only occur on previously vacant land at the city edge the conclusions regarding determinants of housing supply elasticity in this section apply to both long and short-run supply models. Supply elasticity falls with city size and transportation cost and rises with the agricultural reservation price and cost of structure inputs but does not depend directly on the Λ, α, or t parameters. Furthermore, supply elasticity falls with the distance from the city center at which the rental price of housing is measured.

V. Housing Supply Elasticity in a Neoclassical City

V-1. Long-run (putty-putty) Housing Supply Elasticity

In a neoclassical model, long-run (putty-putty) supply elasticity differs from short-run (putty-clay) elasticity because in the long run housing price increases result in additional housing space construction throughout the city. Indeed, vacant land conversion may constitute a small portion of the long-run supply response. Given the durability of housing, this process may take several decades which creates significant empirical issues that need not be confronted in a theoretical model.

Also, in a neoclassical model, households substitute away from housing consumption when its relative price rises and the quantity of housing space per household falls.[8] Thus supply elasticity of space differs from supply elasticity of housing units. Results for both long-run elasticities in a putty-putty are developed here. This is a theoretical exercise. It does not claim to determine the time frame needed to achieve a long versus short-run equilibrium adjustment of the housing market. Indeed, given that housing prices are changing continuously, it is not clear that a long-run equilibrium of the housing market is ever observed empirically.

The setup here follows Brueckner (1987). Developers produce housing according to a neoclassical production function, $H = F(K, l)$, where K and l are structure and land inputs respectively, H is housing space produced and $F(.)$ is concave in inputs with constant returns to scale. The developer's problem is to choose inputs, K and l, to maximize profit:

$$Max_{K,l} \quad p_k F(K, l) - iK - r_k l \qquad (V\text{-}1)$$

where p_k is the rental price per unit housing space at distance k from the CBD, i is the exogenous market price of structure inputs, and r_k is land rent. Land is assumed owned by absentee landlords. Entry of developers drives their economic profit to zero as in the classical model.

Given constant returns to scale, it is possible to write the developer's problem in terms of profit per unit land and housing production in terms of the $s = K/l$ ratio. Housing production per unit land is given by $h(s) = F(s, 1)$, where $h'(s) > 0$ and $h''(s) < 0$. The developer's problem is now to maximize $p_k h(s) - is - r_k$. First-order conditions for zero profit equilibrium yield:

$$p_k \partial h/\partial s = i \qquad (V\text{-}2)$$

[8] The analysis only concerns consumption of a primary residence. Consumption of a secondary residence is considered part of the composite commodity, having nothing to do with the local housing market being modeled.

$$p_k\, h(s) = is + r_k \qquad (V\text{-}3)$$

This system yields solutions for optimal structure to land ratio at any location, $s_k = S(p_k, i)$ and land rent $r = R(p_k, i)$.

Proposition III: Supply elasticity is not determined by the fraction of land available for housing production, Λ, which may reflect factors such as topography and building regulations that limit the fraction of land available for housing.

Total housing production at a given distance, *k*, is given by

$$H_k = 2\pi\Lambda k\, h(s_k) = 2\pi\Lambda k\, h(S(p_k, i))) \qquad (V\text{-}4)$$

It follows that, as shown for the classical model and equations *(IV-9 through IV-10')*, this elasticity is not a function of Λ, *i.e.* $\partial\, \epsilon_{|k}/\partial\Lambda = 0$. Here Λ can be interpreted as the fraction of land available for development due to topography, regulation, or preemption by non-residential land uses. As with the classical model, this elasticity is not a function of Λ, *i.e.* $\partial\, \epsilon_{|k}/\partial\Lambda = 0$. *Proposition III* is a restatement of a portion of *Proposition I* and demonstrates that the lack of a relation between Λ and long-run supply elasticity holds for neoclassical as well as classical models as it relies on a fundamental geometric property of the urban land market.

For given *k* the price elasticity of housing supply is

$$\epsilon_{|k} = [\partial H_k/\partial p_k][p_k/H_k] = [\partial \log h(S(p_k, i))/\partial p_k]\, p_k \qquad (V\text{-}5)$$

Because this is a neoclassical model, the elasticity of housing space supply is not equal to the elasticity of housing unit supply. Households respond to rising price by consuming less space. Let h_k be housing space per household at distance *k*. Then households at that distance are given by $N_k = H_k/h_k$ and changes in this household count are related to changes in housing space by:

$$dN_k = [\partial(H_k/h_k)/\partial H_k]\, dH_k + [\partial(H_k/h_k)/\partial h_k]\, dh_k \qquad (V\text{-}6)$$

The effect of house price on households is:

$$dN_k/dp_k = [(1/h_k)\, \partial(H_k/\partial p_k)] - [(H_k/h_k^2)[\partial h_k/\partial p_k)] \qquad (V\text{-}6')$$

Therefore the elasticity of supply of housing units can be written as:

$$E_k = \{[(1/h_k)\, \partial(H_k/\partial p_k)] - [(H_k/h_k^2)[\partial h_k/\partial p_k)]\}\, [p_k/(H_k/h_k)] \quad or$$

$$E_k = [\partial(H_k/\partial p_k)(p_k/H_k)] - [\partial h_k/\partial p_k)](p_k/h_k) \qquad or$$

$$E_k = \epsilon_k - [\partial h_k/\partial p_k)](p_k/h_k) = \epsilon_k - \varepsilon_k \qquad (V\text{-}6'')$$

Where $\varepsilon_k < 0$ is the own price elasticity of household demand for space at k. Clearly $E_k > \epsilon_k > 0.$

Now consider total city housing space supply from $k = k_o$ to the city boundary at k^* where the rental price of urban land falls to the agricultural reservation price, $r_A = R(p_{k^*}, i, k^*)$. Total housing production is

$$Q = \int_o^{k^*} 2\pi\Lambda\, h(S(P(k), i)\, dk \qquad (V\text{-}7)$$

Let $p^@$ be the average price of housing in the city and hence it is the price at which the average density of housing is produced $h^@ = h(S(p^@, i)$.[9] With this definition, city space supply elasticity can be written as:

$$\epsilon = [\partial Q/\partial p^@]\, [p^@/Q] = [\partial \log h(S(p^@, i)/\partial\, dp^@]\, p^@ \qquad (V\text{-}8)$$

Once again, this expression is not a function of Λ, i.e. $\partial\, \epsilon/\partial\Lambda = 0$. This was also noted for housing supply elasticity in the classical model. Similarly housing unit supply, $E = \epsilon_k - \varepsilon_k$, is not a function of Λ, i.e. $\partial E /\partial\Lambda = 0$. This is in contrast with previous literature which has argued that housing

[9] Proof of the existence of such an average price is given in the appendix.

unit supply elasticity depends on topography, regulation, or other factors influencing the fraction of land available for residential real estate.

Proposition IV: The effect of uniform changes in transportation on the supply elasticity of both interior space and number of units is ambiguous.

This section considers long-run effects of changes in transportation cost on the elasticity of supply of housing space and units and requires explicit modeling of the household location and housing consumption decisions. Homogenous households maximize utility by choosing a composite non-housing good with price equal unity everywhere and housing space per household, *h*, subject to a budget constraint

$$Max_{c,h}\ U(c, h)$$

$$s.t.\ \ y = c + tk + P(k)h \qquad (V\text{-}9)$$

where *t* is a uniform commuting cost per unit distance, and y is exogenous income earned at the city center. Workers employed outside the center at distance *k* earn $y - tk$. Muth's equation derived from the household's problem in *(V-9)* implies that $\partial P(k)/\partial k = -t/h$ and it follows that $\partial P(k)/\partial t < 0$ for all $k \leq k^*$. In equilibrium, Equation *(V-5)*, supply elasticity of housing space at any *k,* can be written:

$$\epsilon_{|k} = [\partial \log h(S(p_k, i))/\partial p_k]\ p_k = [h'(s)/h(s)][\partial s/\partial p_k]\ p_k$$

$$= [i/h(s)][\partial s/\partial p_k] \qquad (V\text{-}10)$$

where the third equality follows directly from the first-order conditions for the developer's optimal choice.

Finally, to determine the effects of variation in transportation cost on elasticity, differentiate Equation *(V-10)* with respect to transportation cost.[10] With some algebraic manipulation the derivative of housing space elasticity at any given distance with respect to transportation cost can be written as:

$$\partial \epsilon_{|k}/\partial t = [i/h(s)][\partial p_k/\partial t] \{[h'(s)/h(s)][\partial s/\partial p_k]^2 - [\partial^2/s/\partial p_k^2]\} \qquad (V\text{-}11)$$

The product, $[i/h(S)][\partial p_k/\partial t]$, is clearly < 0. Therefore the expression in brackets, { }, determines how the elasticity of supply at any given distance varies with transportation cost. The first term of the expression in brackets is negative, $[h'(S)/h(S)][\partial S/\partial p_k]^2 < 0$. Under the usual assumptions regarding the housing production function, $[\partial^2/s/\partial p_k^2] < 0$ and the effect of increasing transportation cost on the elasticity of housing space supply at any distance is not determinant without further insight into the specific supply function.

Effects of transportation cost on the elasticity of supply of housing units follow from (V-6") by subtracting own price elasticity of demand from the expression for $\epsilon_{|k}$ in (V-10):

$$E_{|k} = \epsilon_{|k} - \varepsilon_{|k} = [i/h(s_k)]\,[\partial s_k/\partial p_k] - [\partial h_k/\partial p_k](p_k/h_k) \qquad (V\text{-}12)$$

The derivative of housing unit supply elasticity at any distance *k* with respect to the transportation cost parameter is given by:

$$dE_{|k}/dt = [i/h(s_k)][\partial p_k/\partial t] \{[h'(s_k)/h(s_k)][\partial s_k/\partial p_k]^2 - [\partial^2/s_k/\partial p_k^2]\}$$

$$- [\partial h_k/\partial p_k]\{(\partial p_k/\partial t)/h_k - (p_k/h_k^2)(dh_k/dt)\} \qquad (V\text{-}13)$$

[10] Transportation cost enters the supply elasticity equation through its effect on the consumer's optimization problem, i.e. through Muth's equation.

The first expression in brackets is just $d\epsilon_{|k}/dt$ whose sign was shown above to be ambiguous. Given that $[\partial h_k/\partial p_k] < 0$, $(\partial p_k/\partial t) < 0$, and $(dh_k/dt) > 0$, the entire expression $[\partial h_k/\partial p_k]\{(\partial p_k/\partial t)/h_k - (p_k/h_k^2)(dh_k/dt)\} > 0$ and, it follows that $dE_{|k}/dt < d\epsilon_{|k}/dt$. As expected, the effects of transportation cost on the elasticity of housing unit supply are not as large as those on housing space supply but the sign of the effect cannot be determined without constraining the form of the supply function.

The discussion of the effects of transportation cost on the overall elasticity of supply can be extended to the elasticity of supply for the entire city. As was the case in the previous section, simply apply the argument in which p_k is replaced by average price, $p^{@}$.

Proposition V: Even if the elasticity of supply of housing space at a given location is constant, and transportation cost per unit distance is constant, long-run supply elasticity varies by location and housing price is not iso-elastic unless the income elasticity of housing demand is zero.

Assume that the housing production function has the property that the elasticity of housing supply at any location is constant and equal to ψ, so that $d\log h/d\log p = \psi$. It follows that, if housing demand is iso-elastic, then the housing price profile of the city will be iso-elastic, as assumed in Cosman, et al. (2018) because $d\epsilon/dk = (d\log h/d\log p)(d\log p/dk) = \psi(d\log p/dk)$. However, even assuming the simplest constant transportation cost function in which cost per unit distance is constant and equal to t, Muth's equation implies that $dp/dk = -t/q$ or that $d\log p/dk = -t/pq$. Given these two conditions it follows that:

$$d\epsilon/dk = (d\log h/d\log p)(d\log p/dk) = \psi(-t/pq) \qquad (V\text{-}14)$$

Therefore, for supply elasticity to be invariant across locations, i.e. for $d\epsilon/dk = 0$, pq must remain constant as $y - tk$ falls with distance or the income elasticity of housing demand must be equal to zero and *Proposition V* is proved. In practice it is likely that both marginal transportation cost and total housing expenditure are decreasing functions and this makes the effect of distance on supply elasticity ambiguous but the point of the proposition is that it is certainly not constant.

In sum, the neoclassical model of an urban housing market implies that long-run housing supply is not a function of the constant fraction of land available for housing unless, it is a decreasing function of transportation cost, and finally that supply elasticity varies with location in the city.

V-2. Short-run (putty-clay) Supply Elasticity in a Neoclassical City

The short-run, putty-clay, response of housing in a neoclassical city is similar to the classical city because developers are only able to influence supply on the vacant land at the edge of the city. Because the model assumes that all land available for housing is developed at the city limit before the city expands, the theory diverges from short-run reality in which some sites are withheld from development as the city expands for reasons outlined in Capozza and Helsley (1990) and subsequent literature. Also, the short-run supply response does not include changing size of existing units in response to the price change.

Finally, the density of housing added in each successive annulus as the price of housing rises, is identical because the developer's optimal solution at the city edge is always identical. This means that the entirety of *Proposition I*, applies, except that the parameter "α" is interpreted as the structure land ratio at the city edge.

Proposition VI: In the short run of a neoclassical (putty-clay) urban land market the elasticity of supply of housing space and number of units in the short run varies inversely with city population and transportation cost and directly with cost of agricultural land and structure inputs. Supply elasticity is not determined by the parameter Λ which represents factors such *as topography and building regulations that limit the fraction of land available for housing or the density of housing units.*

Short-run supply elasticity depends only on the amount of land added at the city boundary as housing price rises based on the same arguments made for the classical model.

VI. Conclusions and Implications

Twenty-five years have passed since the publication of "Why Don't We Know More About Housing Supply?" (DiPasquale, 1999), which focused attention on the empirical challenges involved in estimating the determinants of housing supply. This paper argues that little is still known about the elasticity of housing supply, of either units or space, in cities and explains why reliable estimation of supply elasticity remains elusive. Several main findings are offered.

First, there is a lack of data on measures of elasticity suggested by theory, i.e. housing space, adjusted for quality, and rental prices. Estimates of the elasticity of supply of units using long-run differences over time ignore dramatic changes in demand for and character of housing. Second, in a growing city, the percentage changes in housing price are expected to vary by location which means that even if percentage changes in housing supply can be measured, measuring the associated price change is problematic. Third, measures of transportation cost, an important and deeply endogenous determinant of supply response, are not available. Fourth, if planning regulations or topographic barriers are uniformly distributed over the city, they should have no effect on supply elasticity. Alternatively, if they are differentially binding across the city, the relation between average housing price and changes in space or unit supply will be distorted compared to a laissez faire response. Taken together, these results suggest ways that estimates of long-run housing supply elasticity can be made more reliably.

The challenges involved in directly estimating the determinants of differences in housing supply are clearly formidable. In contrast, carefully calibrated numerical urban simulation models can provide results for individual cities and produce estimates of the elasticity of supply or units or interior space that are consistent with theory. These measures are not compromised by changes over time in housing demand. Alternatively measures of the urban wage premium for cities can potentially provide general measures of housing supply elasticity and perhaps uncover some determinants.

In sum, theory has major implications for attempts to explain differences in estimated supply elasticity among cities. Some factors, such as topographic barriers and restrictions on height and density, which have received significant attention in the literature, are shown here to have a problematic relation to supply elasticity, unless they are distributed systematically over space. Furthermore, a list of factors that are difficult to measure, including transportation cost and value of land at the urban edge, are consequential for supply elasticity. This means that it is easy to confuse the importance of various factors influencing housing supply elasticity. Finally, there is a tendency for supply elasticity to fall with city size that confounds attempts to measure determinants of differences in housing supply in a cross section or panel of cities.

The elasticity of housing supply in cities is consequential because it determines the allocation of labor among more or less productive locations. However, the results in this paper suggest that researchers should exercise caution in calling for policies to address low housing supply elasticity, based on empirical estimates of determinants of the relation between city characteristics and measured supply elasticity.

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