

Lecture 4:

Surplus and Consumer Choice

September 15, 2026

Course Administration 1 of 2 – Feedback

Quiz today

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Quiz today

PS 2

- ① graded, I believe!
- ② Answers posted
- ③ I will generally post PS answers – please nudge if I forget
 - revising answer to Q21 to include (a) and (d)
 - if you have concerns on grading, write TA Tanya
- ④ Put your name on the assignment

Course Administration 2 of 2 – Course Progress

- ① Happy to see you in office hours – use scheduler
- ② Midterm is Week 8; this is Week 4
- ③ 2025 midterm posted at very bottom of lectures page
 - will not post answers, but happy to discuss
- ④ Any questions or outstanding issues?

Ripped From the Headlines

By **midnight this Wednesday**, finders need to send an article related to either markets and policy or today's lecture.

Afternoon

Finder	Grapher
Michelle	Elizabeth
Bora	Jimmy
	Bridget

Bridget: choose the article you prefer!

Evening

Finder	Grapher
Karolina	Sophia
Fatima	Jewel
Olivia	Emily G.

Plan for Today: Consumer Behavior

- ① Why we do this
- ② Utility
- ③ Indifference curves
- ④ Budget constraint
- ⑤ Putting it all together

Chapter 4: Individual Demand

Now – Chapter 4: Shift to Consumers' Problem

- Until now
 - Supply and demand equilibria
 - And policy choices that impact equilibrium outcomes
 - Graphs with P and Q axes

Now – Chapter 4: Shift to Consumers' Problem

- Until now
 - Supply and demand equilibria
 - And policy choices that impact equilibrium outcomes
 - Graphs with P and Q axes
- Next two classes focus on how economists think about consumer choices
- Today
 - Lay out consumer decision
 - Graphs with good X and good Y axes
- Lecture 5
 - Use consumer framework to understand responses to price changes and income changes

Why Do We Study the Consumer's Problem?

- Build up to the demand curve from first principles
- Understand consumer choices
- Clearly illuminate areas where policy can act
- Illustrate welfare consequences of policy choices
- Understand intuition of constrained maximization

Also Learn How Economists Think

- All decisions involve trade-offs
- Problems where agents try to do the best they can subject to constraints
- Called constrained minimization or maximization problems

Where We Are Going

- ① Consumer preferences and utility
- ② Indifference curves
- ③ Budget constraints
- ④ Optimization

Utility

Assumptions about Consumer Preferences

① Completeness and Rankability

- You can rank all your consumption choices
- For two bundles A and B , you always either
 - prefer A to B
 - prefer B to A
 - are indifferent between A and B

Assumptions about Consumer Preferences

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- If A is preferred to B , and B to C , then $A > C$

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- If A is preferred to B , and B to C , then $A > C$

④ The more you have of a particular good, the less of something else you are willing to give up to get more of that good

What is Utility?

Overall satisfaction or happiness

What is Utility?

Overall satisfaction or happiness

- Measured in utils!
- This framework allows us to describe what consumption or habits make you happier than other consumptions or habits
- It's not a tool for comparing across people

Some Example Utility Functions

Most general $U = U(X, Y)$

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They can take many forms, such as

- $U = U(X, Y) = XY$
- $U = U(X, Y) = X + Y$
- $U = U(X, Y) = X^{0.7} Y^{0.3}$

Marginal Utility

Marginal utility $\equiv$ “additional utility consumer receives from an additional unit of a good or service”

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$$MU_X = \frac{\Delta U(X, Y)}{\Delta X} \left(= \frac{\partial U}{\partial X} \right)$$
$$MU_Y = \frac{\Delta U(X, Y)}{\Delta Y} \left(= \frac{\partial U}{\partial Y} \right)$$

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Following our assumptions, what is generally true about marginal utility of X as consumption of X increases?

Basics of Marginal Utility

Coffee

- Your first cup of coffee: 3 utils
- Your second cup of coffee: 2 utils
- What is the marginal utility of the second cup?

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- You had 64 GB of memory, giving 10 utils
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- 10 additional utils

Utility and Comparisons

- Ordinal: we can rank bundles from best to worst
- Not cardinal: we cannot say how much one bundle is preferred to another in fixed units
- We cannot make interpersonal comparisons

No other assumptions on utility apart from the four preference assumptions.

Indifference Curves

Indifference Curves, or What You Like

Indifference Curve Plan

- ① What they are
- ② Why they have the shape they do
- ③ Math for describing the trade-offs indifference curves describe
- ④ Substitutes and complements

Describing Your Utility

- Let i denote a bundle of goods, so $i \in \{1, 2, 3, \dots\}$
- An indifference curve is a set of bundles (X_i, Y_i) where $U(X_i, Y_i)$ is the same for any i

Describing Your Utility

- Let i denote a bundle of goods, so $i \in \{1, 2, 3, \dots\}$
- An indifference curve is a set of bundles (X_i, Y_i) where $U(X_i, Y_i)$ is the same for any i
- Suppose a consumer has two bundles, (X_1, Y_1) and (X_2, Y_2)
- Consumer is indifferent when $U(X_1, Y_1) = U(X_2, Y_2)$

Working Up to an Indifference Curve

- Give me two items

Working Up to an Indifference Curve

- Give me two items
- Each axis is a **quantity** of those items
- Give me some points where you are equally happy

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Why Can We Draw Indifference Curves?

- Because of the assumptions we made at the beginning about preferences: completeness and rankability
- All bundles have a utility level and we can rank them

Indifference Curves Level and Slope

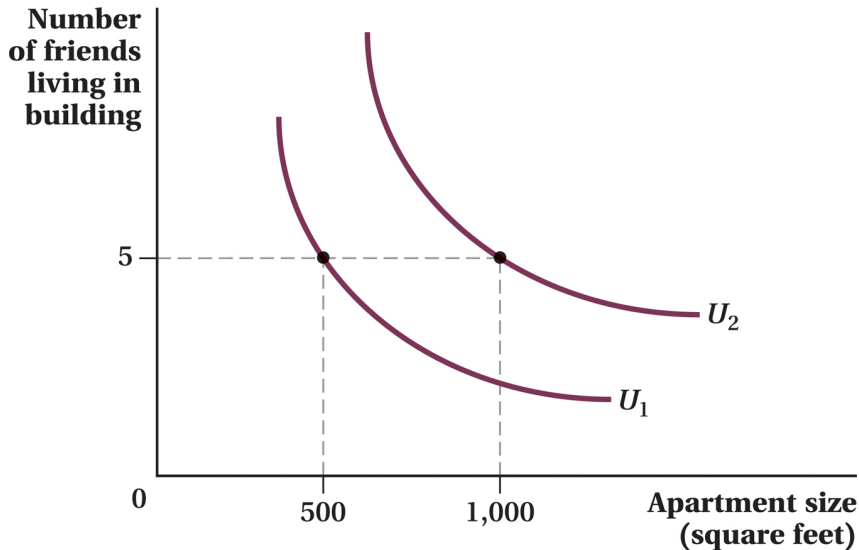
What does “more is better” tell us?

Indifference Curves Level and Slope

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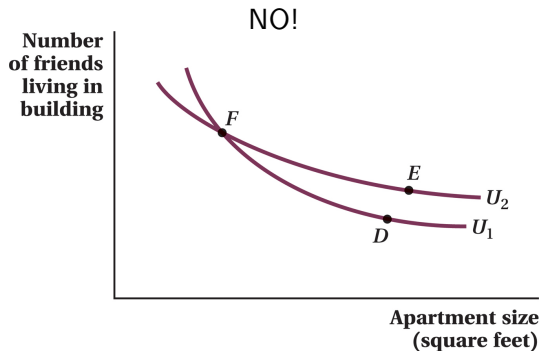
- That higher indifference curves give more utility
- Curve must have a negative slope
 - Suppose that you increase your consumption of X
 - “More is better” $\rightarrow$ you are happier
 - To be equally happy as before, you should give up some Y

More Utility on Curves Farther From Origin



Indifference Curve Shape

- Curves never cross
 - it would violate transitivity
- Curves are U-like (convex) with respect to the origin
 - Comes from assumption about diminishing marginal utility
 - Your willingness to trade off differs along the curve

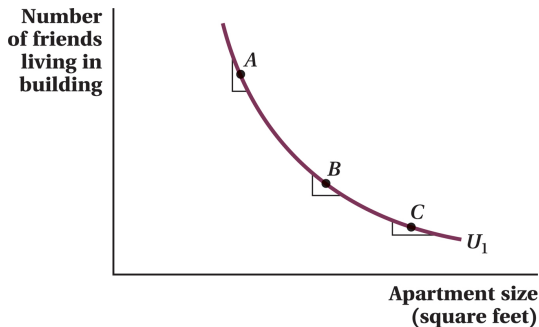


Steepness of the Indifference Curve

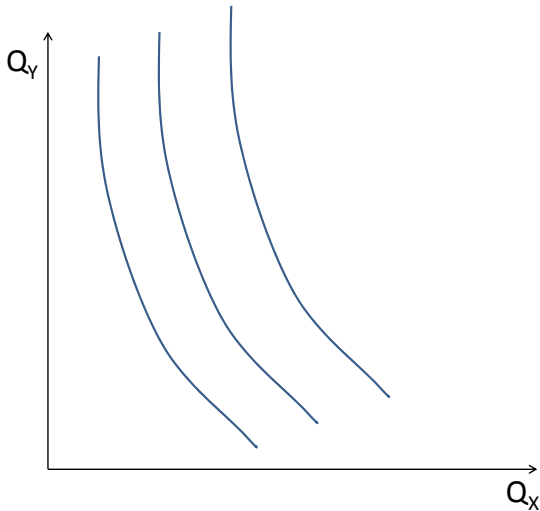
- We know that you are equally happy anywhere along the indifference curve
- So what changes as you move along the curve?

Steepness of the Indifference Curve

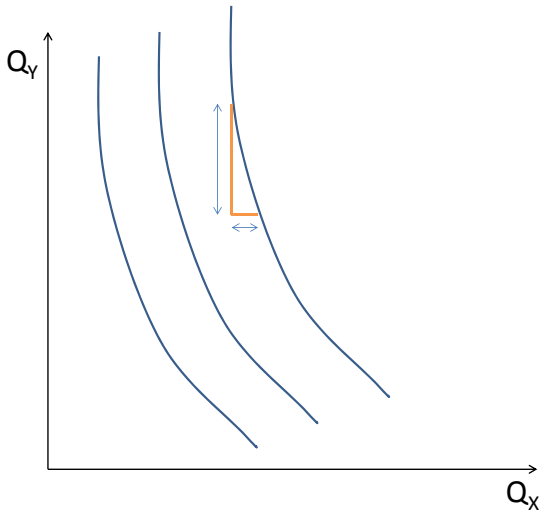
- We know that you are equally happy anywhere along the indifference curve
- So what changes as you move along the curve?
 - you are trading off X and Y
 - the rate at which you trade them off tells us how much you value them



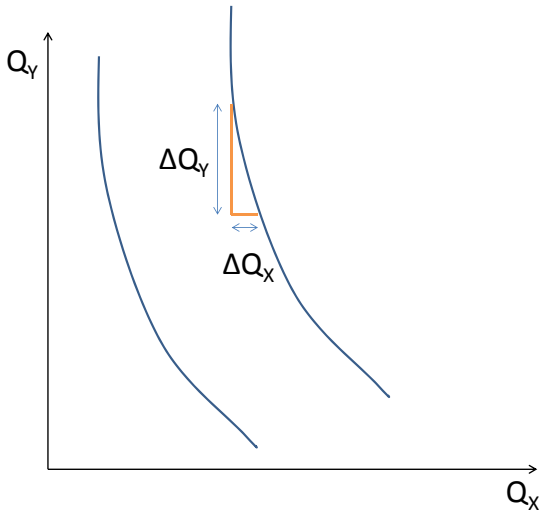
When the curve is steep, what are you willing to give up more of?



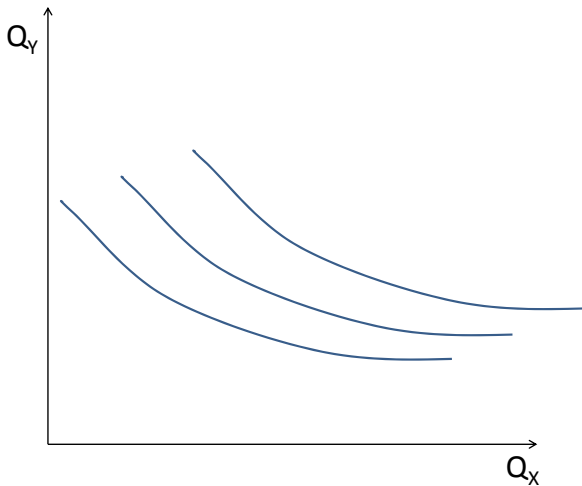
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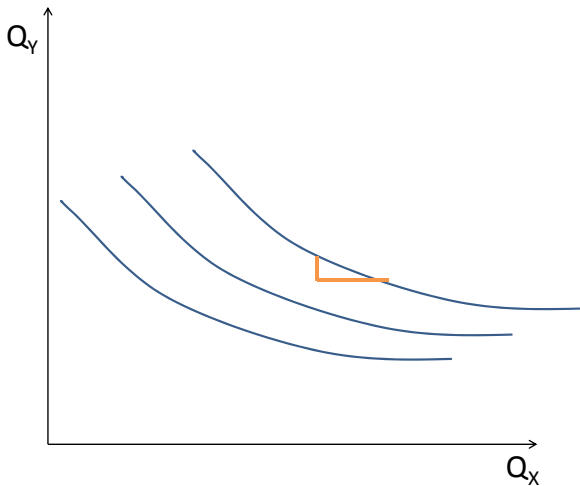
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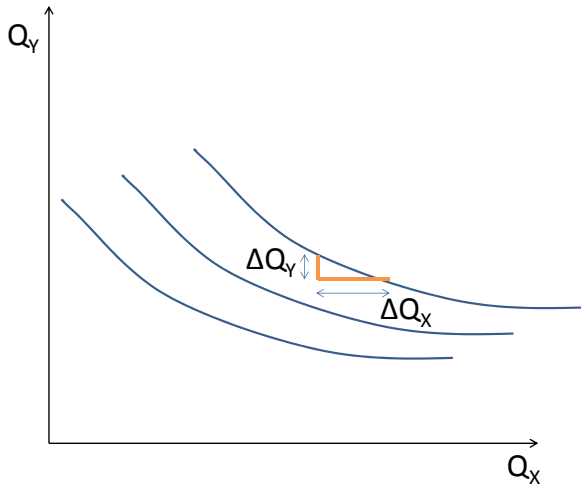
When the curve is flat, what are you willing to give up more of?



When the curve is flat, what are you willing to give up more of?



When the curve is flat, what are you willing to give up more of?



Quantifying the Trade-off in the Indifference Curve

- How much of X are you willing to give up for Y ?
- **Marginal Rate of Substitution** is the trade-off
- Define

$$MRS_{XY} = (-1) * \frac{MU_X}{MU_Y}$$

MRS_{XY} = slope of indifference curve

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- Given shape of the indifference curve, MRS is negative
- Is it the same everywhere on the curve?

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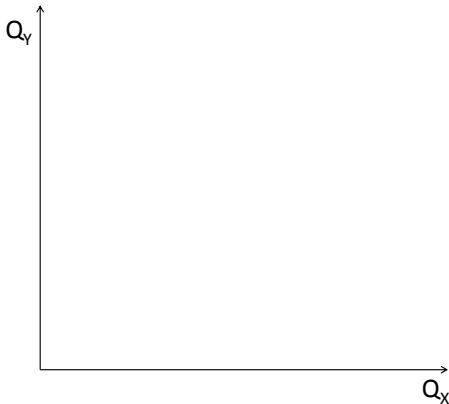
MRS_{XY} = slope of indifference curve

- A rate of change along the indifference curve
- Given shape of the indifference curve, MRS is negative
- Is it the same everywhere on the curve? Not necessarily.
- If you want a derivation, see the textbook!

Indifference Curves for Perfect Complements

Work with your neighbor!

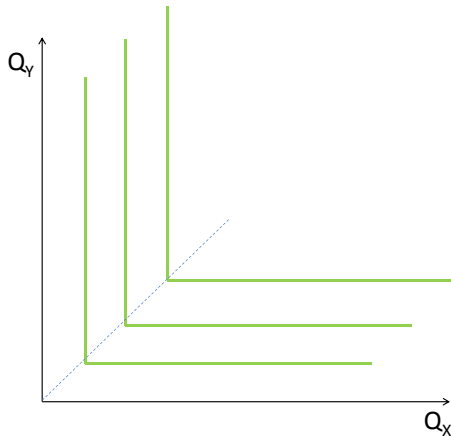
- Suppose we have two goods that are perfect complements
- X and Y being perfect complements means each is useless without the other
- What do the indifference curves look like?
- We write this utility as
$$U = \min\{aX, bY\}$$



Indifference Curves for Perfect Complements

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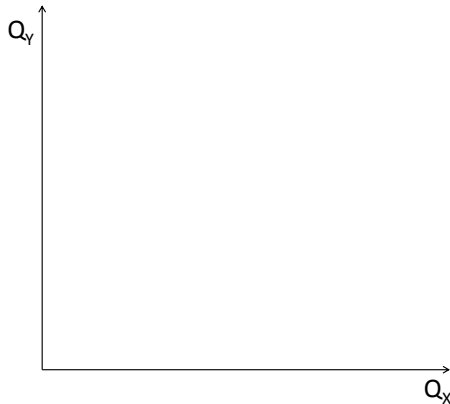
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Indifference Curves for Substitutes

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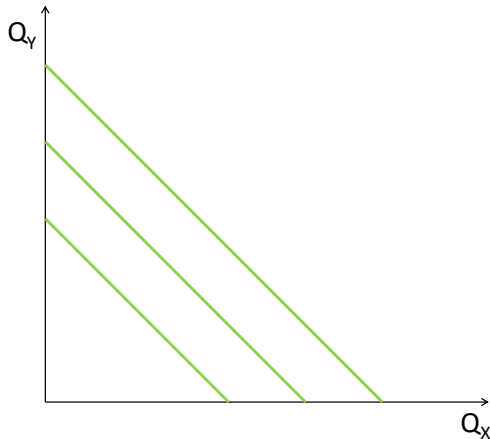
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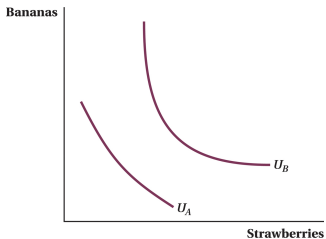
Indifference Curves for Substitutes

Work with your neighbor!

- Suppose we have two goods that are perfect substitutes
- What do the indifference curves look like?
- Write as $U = aX + bY$



Curves May Change Shape as Consumption Increases



- the same person can have differently shaped indifference curves
- at different levels of happiness
- U_A : low happiness – willing to make any trade-offs
- U_B : high happiness – less willing to trade off

How Algebra Tells Us About Substitutes and Complements

In which equation(s) are X and Y substitutable? in which are they complementary?

- $U = U(X, Y) = XY$
- $U = U(X, Y) = X + Y$
- $U = U(X, Y) = X^{0.7} Y^{0.3}$

Budget Constraint

Budget Constraint, or What You Can Afford

Budget Constraint Plan

- ① Define
- ② Draw
- ③ Find slope

Budget Constraint Assumptions

- ① Each good has a fixed price and infinite supply
- ② Each consumer has a fixed amount of income to spend
- ③ The consumer cannot save or borrow

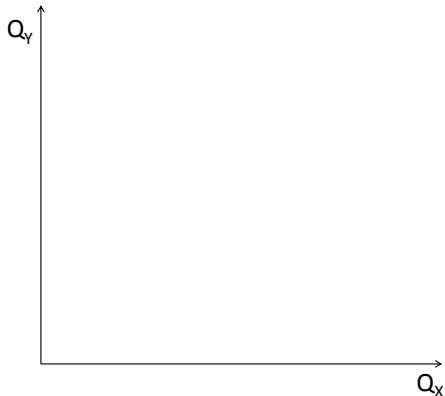
Defining the Budget Constraint

Budget constraint:

$$I = P_X Q_X + P_Y Q_Y$$

- **feasible bundle** $\equiv$ combinations of X and Y that the consumer can purchase with his income
- **infeasible bundle** $\equiv$ all the combinations the consumer is just too poor to get

Drawing the Budget Constraint

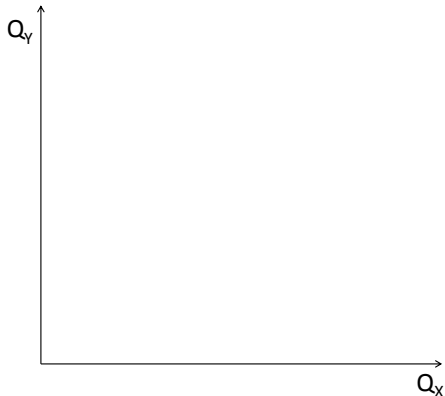


What if you spend all your money on X or all your money on Y ?

- If you spend it all on X

$$I = P_X Q_X + P_Y Q_Y$$

Drawing the Budget Constraint



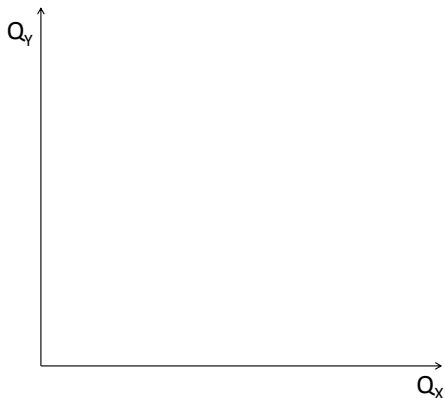
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$$I = P_X Q_X + P_Y (0)$$

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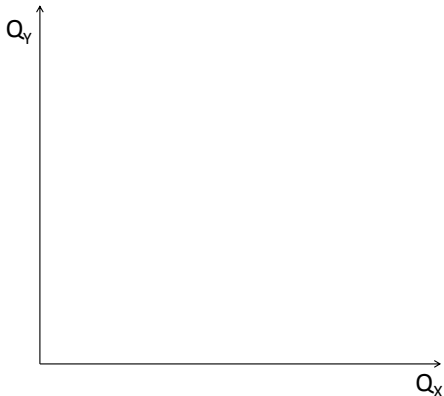
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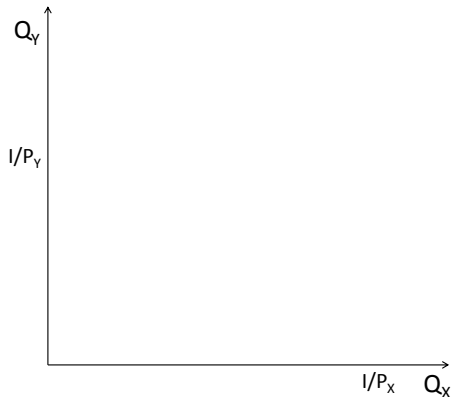
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$$I = P_X Q_X$$

$$Q_X = \frac{I}{P_X}$$

Drawing the Budget Constraint



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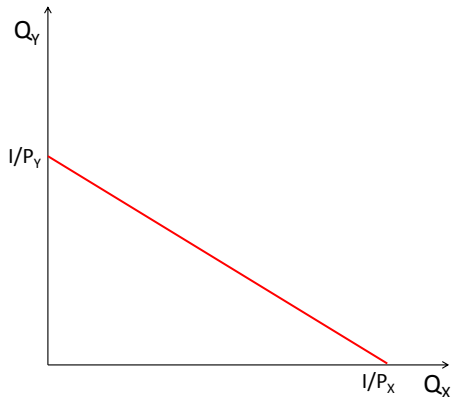
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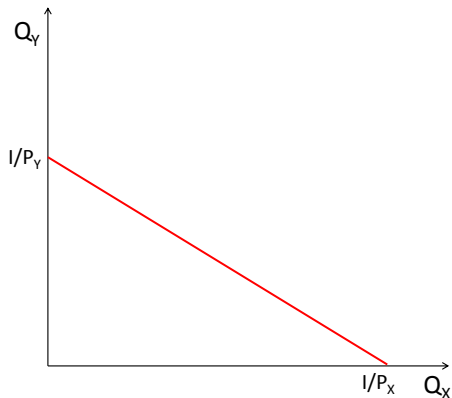
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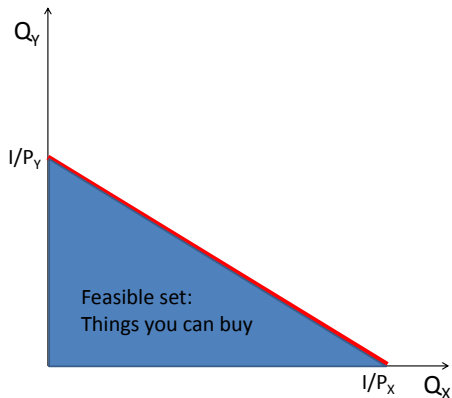
Drawing the Budget Constraint



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What points are feasible to purchase?

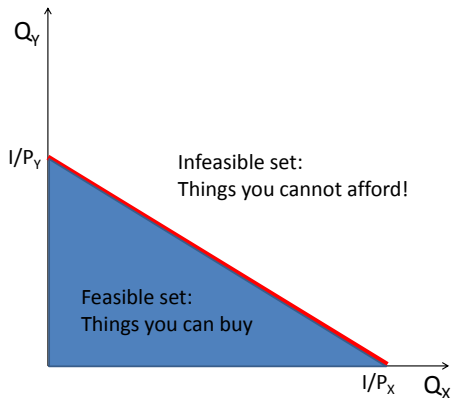
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Drawing the Budget Constraint



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Slope of the Budget Constraint

Algebra of the slope: Write $Q_Y = f(Q_X)$

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Algebra of the slope: Write $Q_Y = f(Q_X)$

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$$P_Y Q_Y = I - P_X Q_X$$

Slope of the Budget Constraint

Algebra of the slope: Write $Q_Y = f(Q_X)$

$$\begin{aligned}I &= P_X Q_X + P_Y Q_Y \\P_Y Q_Y &= I - P_X Q_X \\Q_Y &= \frac{I}{P_Y} - \frac{P_X Q_X}{P_Y}\end{aligned}$$

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$$Q_Y = -\frac{P_X}{P_Y} Q_X + \frac{I}{P_Y}$$

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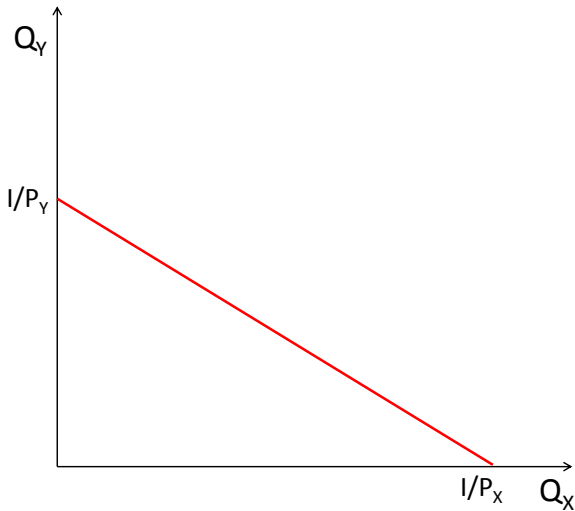
So an additional unit of Q_X requires you to give up $\frac{P_X}{P_Y}$ of Q_Y

What Two Things Affect the Position of the Budget Constraint?

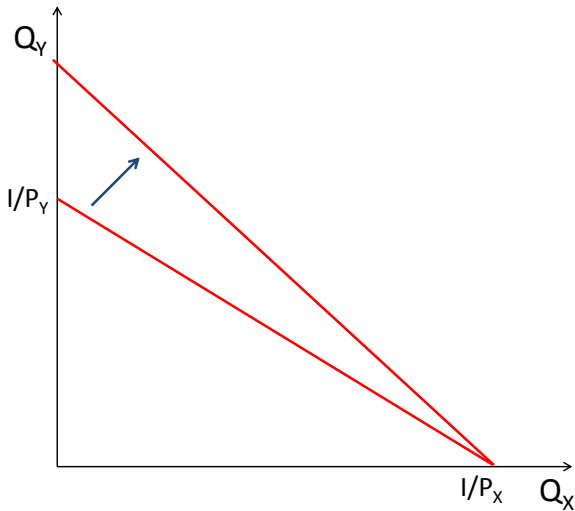
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Prices and Income

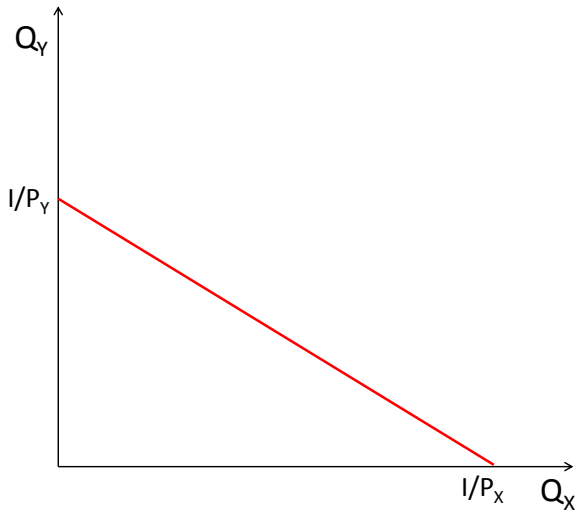
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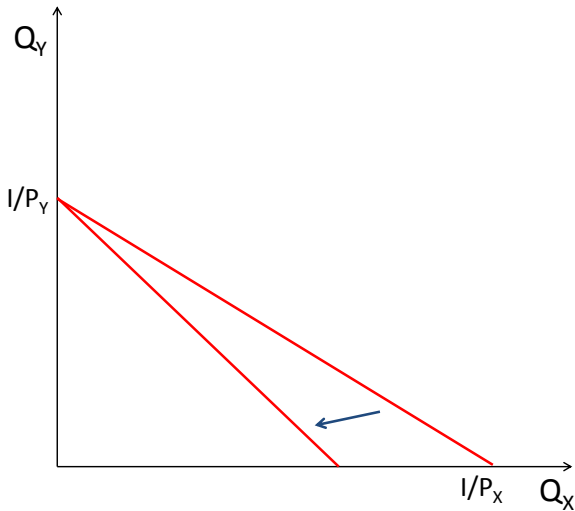
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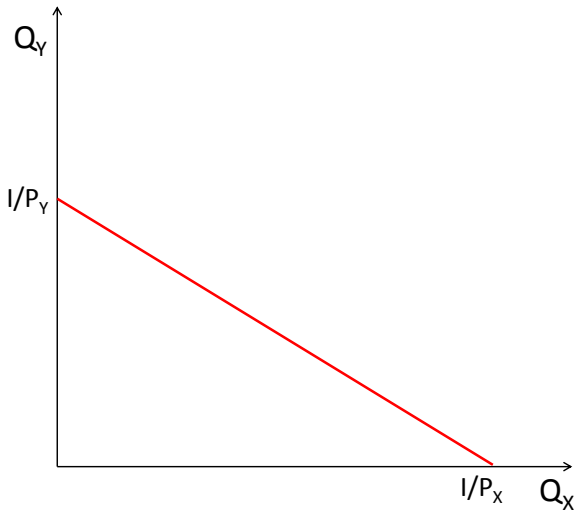
What Happens if the Price of X Increases?



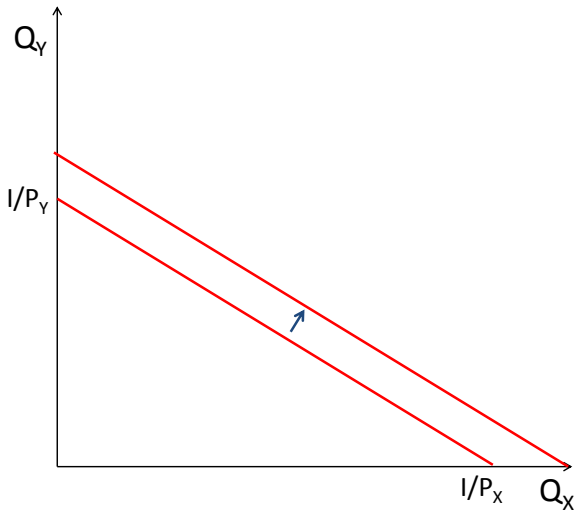
What Happens if the Price of X Increases?



What Happens if Income Increases?



What Happens if Income Increases?



Budget Constraint Changes, In Sum

- Things that change the slope
 - Change in prices, P_X or P_Y
- Things that don't change the slope, but move the line in and out
 - Change in income

Optimizing

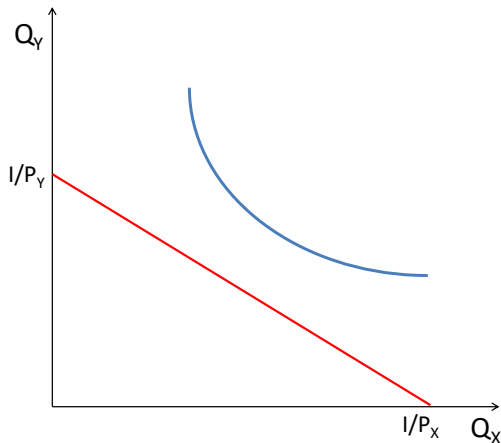
Optimizing, or
The Best You Can Do, Given What You Like
and What You Can Afford

How to Be As Happy as Possible

- Maximize your utility given your budget constraint
- Can this person get the level of utility drawn on the indifference curve given his budget constraint?

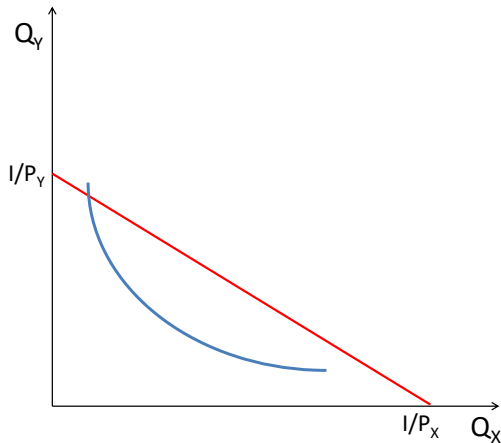
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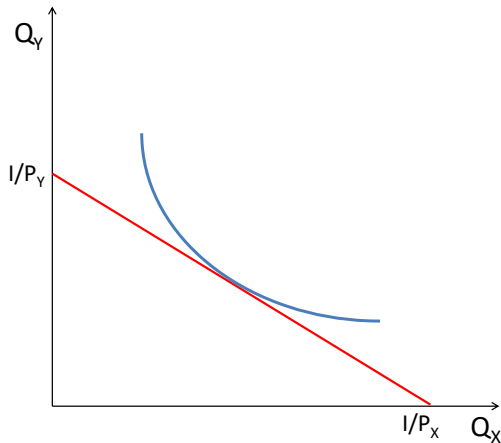
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Algebra of Utility Maximization

- Utility is maximized, given the budget constraint, when the slope of the indifference curve is tangent to the budget constraint
- tangency $\rightarrow$ equality

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$$-\frac{MU_X}{MU_Y} = -\frac{P_X}{P_Y}$$

Algebra of Utility Maximization

- Utility is maximized, given the budget constraint, when the slope of the indifference curve is tangent to the budget constraint
- tangency $\rightarrow$ equality

$$\begin{aligned}MRS_{XY} &= -\frac{P_X}{P_Y} \\ -\frac{MU_X}{MU_Y} &= -\frac{P_X}{P_Y} \\ \frac{MU_X}{P_X} &= \frac{MU_Y}{P_Y}\end{aligned}$$

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- One of the few formulae you need to memorize

When Are You Optimizing?

By definition

- if $MRS_{XY} = P_X/P_Y$ you are optimizing
- if $MRS_{XY} \neq P_X/P_Y$ you are **not** optimizing*

* unless you are at a corner solution, where you consume only one good

How to Find Optimal Consumption?

Start with two equations and two unknowns

- Budget constraint

$$I = P_X X + P_Y Y$$

- Optimality condition

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

MU_X and MU_Y are in terms of X and Y

This is enough to find optimal consumption of X and Y .

In-Class Problem

Set-up

- Sarah gets utility from soda (S) and hotdogs (H)
- Her utility function is $U = S^{0.5}H^{0.5}$
- Therefore
 - $MU_S = 0.5 \frac{H^{0.5}}{S^{0.5}}$
 - $MU_H = 0.5 \frac{S^{0.5}}{H^{0.5}}$
- Sarah's income is \$12
- Price of soda is \$2
- Price of hotdogs is \$3

Questions

- ① Write the equation for Sarah's budget constraint
- ② Draw Sarah's budget constraint
- ③ Write the marginal rate of substitution in terms of H and S
- ④ What amount of sodas and hotdogs makes Sarah happiest, given her budget constraint?
 - Recall that you have two equations and two unknowns
 - budget constraint
 - $MRS = \text{ratio of prices}$

Answer to In-Class Problem, 1 of 2

- We have two unknowns (optimal H and S), so we need two equations to solve for them
- Equation 1: From the information about income and prices, we can write a budget constraint

$$\begin{aligned} I &= P_S S + P_H H \\ 12 &= 2S + 3H \end{aligned}$$

- Equation 2: We know that at equilibrium $-MRS_{H,S} = -\frac{P_H}{P_S}$, or $-MRS$ is equal to the slope of the budget constraint.

$$\begin{aligned} -MRS_{S,H} &= -\frac{P_S}{P_H} \\ \frac{MU_S}{MU_H} &= \frac{0.5H^{0.5}S^{-0.5}}{0.5S^{0.5}H^{-0.5}} = \frac{2}{3} \\ \frac{H}{S} &= \frac{2}{3} \end{aligned}$$

Answer to In-Class Problem, 2 of 2

Now we have two equations and two unknowns. From Equation 2, we know that $H = \frac{2}{3}S$. We can therefore plug this into the budget constraint:

$$12 = 2S + 3H$$

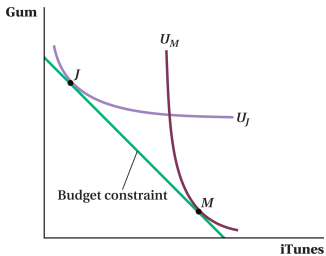
$$12 = 2S + 3\frac{2}{3}S$$

$$12 = 2S + 2S = 4S$$

$$S = 3$$

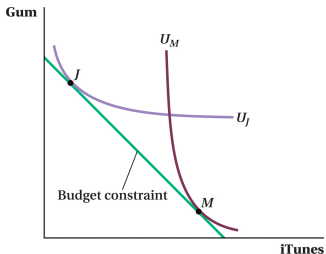
If $S = 3$, then $12 = 2 * 3 + 3H$, which means that $6 = 3H$, or $H = 2$. And Sarah has spent all her money.

A Usual Maximization of Utility s.t. Budget Constraint



- People with different preferences
- But consume at the same MRS

A Usual Maximization of Utility s.t. Budget Constraint



- People with different preferences
- But consume at the same MRS
- Because they face the same prices!
- But don't consume the same thing

What We Did This Class

- ① Preferences and utility
- ② Indifference curves – what you like
- ③ Budget constraint – what you can afford
- ④ Optimization – the best you can do given what you like and can afford

The Big Ideas

- Utility
- What you like: indifference curves
- What you can afford: budget constraint
- The best you can do given these two: optimization

Next Class

- Read Chapter 5
 - Omit income Engel curves from 5.1
 - Skip 5.3 entirely